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Some Fixed Point Results of Non-Newtonian Expansion Mappings

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Authors' contributions

This work was carried out in collaboration between all authors. Authors MU and BP designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BP managed the analyses of the study. Author RDD managed the literature searches. All authors read and approved the final manuscript.

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Abstract

We manifest some fixed point and common fixed point results for non-Newtonian expansive maps defined on non-Newtonian metric spaces. The results offered in this article comprise non-Newtonian metric generalizations of some fixed point results in the literature.

Keywords: Non-Newtonian metric space; non-Newtonian expansive mapping; fixed point.

1. INTRODUCTION

The idea of non-Newtonian calculus was firstly acquaint by Grossman and Katz [1]. Later, the non-Newtonian calculus is studied by Bashirov et al. [2], Ozyapici et al. [3], Cakmak and Basar [4] and others

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[5-17]. Cakmak and Basar [4] have studied the concept of non-Newtonian metric. Several statements about them are proven in [7]. Binbasıoglu et al. [18] defined the contractive mapping in non-Newtonian metric space. The non-Newtonian calculi are alternatives to the classical calculus of Newton and Leibnitz. They confer a wide variety of mathematical tools for usage in technology and mathematics. The non-Newtonian calculus has great applications in various areas including fractal geometry, the economics of climate change, image analysis, physics, quantum physics, growth/decay analysis, finance, the theory of elasticity in economics, marketing and gauge theory, information technology, pathogen counts in treated water, actuarial science, tumor therapy and cancer-chemotherapy in medicine, materials science/engineering, demographics, finite-difference methods, differential equations, averages of functions, calculus of variations, means of two positive numbers, least-squares methods, multivariable calculus, weighted calculus, meta-calculus, approximation theory, probability theory, utility theory, Bayesian analysis, complex analysis, functional analysis, stochastics, chaos theory, dimensional spaces, decision making, dynamical systems etc.

The study of expansive maps is a very enthralling research area in fixed point theory. Wang et al. [19] deputized the concept of expanding maps and vouched some fixed point results in complete metric spaces. Daffer and Kaneko [20] vouched some common fixed point results in complete metric spaces for two expansive mappings. For more details, we refer the reader to [21-26].

In this article, we give some properties of the relevant non-Newtonian metric space and non-Newtonian normed space. We also introduce the concept of non-Newtonian expansive mappings and present some fixed point results in non-Newtonian metric space. These results also generalize some results obtained previously.

2. PRELIMINARIES

An injective function whose domain is $\mathbb{R}$, the set of all real numbers, and whose range is a subset of $\mathbb{R}$ is called a generator. Each generator generates exactly one type of arithmetic, and conversely each type of arithmetic is generated by exactly one generator. As a generator, we choose the function exp from $\mathbb{R}$ to the set $\mathbb{R}^+$ of positive reals, that is to say,

$$\alpha: \mathbb{R} \longrightarrow \mathbb{R}^+,$$
$$r \longmapsto \alpha(r) = e^r = s$$

and

$$\alpha^{-1}: \mathbb{R}^+ \longrightarrow \mathbb{R},$$
$$s \longmapsto \alpha^{-1}(s) = \ln s = r$$

If $I(r) = r$ for all $r \in \mathbb{R}$, then I is called identity function and we know that inverse of the identity function is itself. If $\alpha = I$, then α generates the classical arithmetic and if $\alpha = exp$, then α generates geometrical arithmetic. All concepts of α-arithmetic have similar properties in classical arithmetic. α-zero, α-one and all α-integers are formed as

$$\ldots\ldots, \alpha(-2), \alpha(-1), \alpha(0), \alpha(1), \alpha(2) \ldots\ldots..$$

The α-positive numbers are the numbers $j \in A$ such that $\dot{0} \dot{<} j$ and the α-negative numbers are those for which $j \dot{<} \dot{0}$. The α-zero, $\dot{0}$, and the α-one, $\dot{1}$, turn out to be $\alpha(0)$ and $\alpha(1)$. The α-integers consist of $\dot{0}$ and all the numbers that result by successive α-addition of $\dot{1}$ and $\dot{0}$ and by successive α-subtraction of $\dot{1}$ and $\dot{0}$.

We denote by $\mathbb{R}(N)$ the range of generator α and write $\mathbb{R}(N) = \{\alpha(r) : r \in \mathbb{R}\}$. $\mathbb{R}(N)$ is called Non-Newtonian real line. Non-Newtonian arithmetic operations on $\mathbb{R}(N)$ are represented as follows:

α-addition $\quad i \dot{+} j = \alpha\left(\alpha^{-1}(i) + \alpha^{-1}(j)\right),$

α-subtraction $\quad i \dot{-} j = \alpha\left(\alpha^{-1}(i) - \alpha^{-1}(j)\right)$,

α-multiplication $\quad i \dot{\times} j = \alpha\left(\alpha^{-1}(i) \times \alpha^{-1}(j)\right)$,

α-division $\quad i \dot{/} j = \alpha(\alpha^{-1}(i)/\alpha^{-1}(j))$,

α-order $\quad i \dot{<} j (i \dot{\leq} j) \Leftrightarrow \alpha^{-1}(i) < \alpha^{-1}(j)\left(\alpha^{-1}(i) \leq \alpha^{-1}(j)\right)$,

The α-square of a number $i \in A \subset \mathbb{R}(N)$ is denoted by $i \dot{\times} i = i^{2_N}$. For each α-nonnegative number v, the symbol $\sqrt{i}^N$ will be used to denote $v = \alpha\left(\sqrt{\alpha^{-1}(i)}\right)$ which is the unique α-square is equal to i, which means that $v^{2_N} = i$. Throughout this paper, i^{p_N} denotes the pth non-Newtonian exponent. Thus we have

$$i^{2_N} = i \dot{\times} i = \alpha\left(\alpha^{-1}(i) \times \alpha^{-1}(i)\right) = \alpha([\alpha^{-1}(i)]^2),$$

$$i^{3_N} = i^{2_N} \dot{\times} i = \alpha\left(\alpha^{-1}(i^{2_N}) \times \alpha^{-1}(i)\right)$$

$$= \alpha\left(\alpha^{-1}\left(\alpha\left(\alpha^{-1}(i) \times \alpha^{-1}(i)\right)\right) \times \alpha^{-1}(i)\right) = \alpha([\alpha^{-1}(i)]^3),$$

:
:

$$i^{p_N} = i^{p-1_N} \dot{\times} i = \alpha([\alpha^{-1}(i)]^p)$$

:
:

The α-absolute value of a number $i \in A \subset \mathbb{R}(N)$ is defined as $\alpha(|\alpha^{-1}(i)|)$ and is denoted by $|i|_N$. For each number $i \in A \subset \mathbb{R}(N), \sqrt{i^{2_N}}^N = |i|_N = \alpha(|\alpha^{-1}(i)|)$. In this case,

$$|i|_N = \begin{cases} i, & \text{if } i \dot{>} \dot{0} \\ \dot{0}, & \text{if } i = \dot{0} \\ \dot{0} \dot{-} i, & \text{if } i \dot{<} \dot{0} \end{cases}$$

Also $\mathbb{R}^+(N)$ denotes non-Newtonian positive real numbers and $\mathbb{R}^-(N)$ denotes non-Newtonian negative real numbers. α-intervals are represented by

Closed α-interval $\quad \dot{[} i, j \dot{]} = [i, j]_N = \{s \in \mathbb{R}(N) : \ i \dot{\leq} s \dot{\leq} j\}$
$= \{s \in \mathbb{R}(N) : \ \alpha^{-1}(i) \leq \alpha^{-1}(s) \leq \alpha^{-1}(j)\}$

Open α-interval $\quad \dot{(} i, j \dot{)} = (i, j)_N = \{s \in \mathbb{R}(N) : \ i \dot{<} s \dot{<} j\}$
$= \{s \in \mathbb{R}(N) : \ \alpha^{-1}(i) < \alpha^{-1}(s) < \alpha^{-1}(j)\}$

Likewise semi-closed and semi-open α-intervals can be represented. For the set $\mathbb{R}(N)$ of non-Newtonian real numbers, the binary operations $(\dot{+})$ addition and $(\dot{\times})$ multiplication are defined by

$$\dot{+} \ : \ \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$$
$$(i, j) \longmapsto i \dot{+} j = \alpha\left(\alpha^{-1}(i) + \alpha^{-1}(j)\right)$$

$$\dot{\times} \ : \ \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$$
$$(i, j) \longmapsto i \dot{\times} j = \alpha\left(\alpha^{-1}(i) \times \alpha^{-1}(j)\right).$$

The fundamental properties provided in the classical calculus is provided in non-Newtonian calculus, too.

Lemma 2.1 (see [4]). $(\mathbb{R}(N),\dot{+},\dot{\times})$ is a topologically complete field.

Lemma 2.2 (see [4])$|i \dot{\times} j|_N = |i|_N \dot{\times} |j|_N \ \forall\ i,j \in \mathbb{R}(N)$.

Lemma 2.3 (see [4]) $|i\dot{+}j|_N \dot{\leq} |i|_N\dot{+}|j|_N, \forall\ i,j \in \mathbb{R}(N)$

The non-Newtonian metric spaces provide an alternative to the metric spaces introduced in [4].

Definition 2.4 (see [4]). Let M be a non-empty set and $d_N: M \times M \longrightarrow \mathbb{R}^+(N)$ be a function such that for all $i,j,k \in M$;

(NNN1). $d_N(i,j) = \dot{0} \Leftrightarrow i = j$
(NNN2). $d_N(i,j) = d_N(j,k)$
(NNN3). $d_N(i,j) \dot{\leq} d_N(i,k)\dot{+}d_N(k,j)$.

Then, the map d_N is called non-Newtonian metric and the pair (M, d_N) is called non-Newtonian metric space.

Definition 2.5 (see [4]). Let M be a vector space on $\mathbb{R}(N)$. If a function $\|\,.\,\|_N : M \longrightarrow \mathbb{R}^+(N)$ satisfies the following axioms for all $i,j \in M$ and $\lambda \in \mathbb{R}(N)$:

(NNN1). $\|i\|_N = \dot{0} \Leftrightarrow i = \dot{0}$
(NNN2). $\|\lambda \dot{\times} i\|_N = |\lambda|_N \dot{\times} \|i\|_N$
(NNN3). $\|i\dot{+}j\|_N \dot{\leq} \|i\|_N\dot{+}\|j\|_N$.

Then it is called a non-Newtonian norm on M and the pair $(M, \|\,.\,\|_N)$ is called a non-Newtonian normed space.

Remark 2.6 (see [4]). Here it is easily seen that every non-Newtonian norm $\|\,.\,\|_N$ on M produces a non-Newtonian metric d_N on M given by

$$d_N(i,j) = \|i\dot{-}j\|_N, \forall\ i,j \in M$$

Definition 2.7 (see [4]). (Non-Newtonian convergent sequence) A sequence $\{j_n\}$ in a non-Newtonian metric space (M, d_N) is said to be non-Newtonian convergent if for every given $\epsilon \dot{>} \dot{0}$, there exists an $n_0 = n_0(\epsilon) \in \mathbb{N}$ and $j \in M$ such that $d_N(j_n, j) \dot{<} \epsilon$ for all $n > n_0$ and is denoted by ${}^{N}\lim_{n\longrightarrow+\infty} j_n = j$ or $j_n \xrightarrow{N} j$ as $n \longrightarrow \infty$.

Definition 2.8 (see [4]). (Non-Newtonian Cauchy sequence) A sequence $\{j_n\}$ in a non-Newtonian metric space (M, d_N) is said to be non-Newtonian Cauchy if for every given $\epsilon \dot{>} \dot{0}$, there exists an $n_0 = n_0(\epsilon) \in \mathbb{N}$ such that $d_N(j_n, j_m) \dot{<} \epsilon$ for all $m,n > n_0$.

Definition 2.9 (see [4]). (Non-Newtonian complete metric space) The space M is said to be non-Newtonian complete if every non-Newtonian Cauchy sequence in M converges.

Definition 2.10 (see [4]). (Non-Newtonian bounded) Let (M, d_N) be a non-Newtonian metric space. The space M is said to be non-Newtonian bounded if there is a non-Newtonian constant $\kappa \dot{>} \dot{0}$ such that $d_N(i,j) \dot{\leq} \kappa$ for all $i,j \in M$. The space M is said to be non-Newtonian unbounded if it is not non-Newtonian bounded.

Proposition 2.11 (see [4]). Suppose that the non-Newtonian metric d_N on $\mathbb{R}(N)$ is such that $d_N(i,j) = |i\dot{-}j|_N$ for all $i,j \in \mathbb{R}(N)$, then $(\mathbb{R}(N), d_N)$ is a non-Newtonian metric space.

Lemma 2.12 (see [18]). Let (M, d_N) be a non-Newtonian metric space. Then,

(1). A non-Newtonian convergent sequence in M is non-Newtonian bounded and its non-Newtonian limit is unique.
(2). A non-Newtonian convergent sequence in M is a non-Newtonian Cauchy sequence in M.

From the definition of non-Newtonian Cauchy sequence and Lemma 2.12, we can give the following corollary:

Corollary 2.13 (see [18]) A non-Newtonian Cauchy sequence is non-Newtonian bounded.

Lemma 2.14 (see [18]) Suppose (M, d_N) is a non-Newtonian metric space and $i,j,k \in M$. Then

$$|d_N(i.j)\dot{-}d_N(j,k)|_N \dot{\leq} d_N(i,k)$$

Definition 2.15 Let M be a set and f a map from M to M. A fixed point of f is a solution of the functional equation $f(j) = j, j \in M$. A point $j \in M$ is called common fixed point of two self-mappings f and g on M if $f(j) = g(j) = j$.

Definition 2.16 (see [18]) Suppose (M, d_N) is a non-Newtonian complete metric space. A mapping $f: M \to M$ is called non-Newtonian Lipschitzian if there exists a non-Newtonian number $\delta \in \mathbb{R}(N)$ such that

$$d_N(f(i), f(j)) \dot{\leq} \delta \dot{\times} d_N(i,j), \forall\, i,j \in M.$$

The mapping f is called non-Newtonian contractive if $\delta \dot{<} \dot{1}$.

Binbasıoglu et al. [18] established following result in non-Newtonian metric space.

Theorem 2.17 Let f be a non-Newtonian contraction mapping on a non-Newtonian complete metric space M. Then f has a unique fixed point.

3. Main Results

Now, we give some properties related to non-Newtonian metric spaces and non-Newtonian normed spaces.

Proposition 3.1 The non-Newtonian distance is commutative.

Proof Let i and j be any two non-Newtonian numbers. Then

$$\begin{aligned} |i\dot{-}j|_N &= \alpha(|\alpha^{-1}(i) - \alpha^{-1}(j)|) \\ &= \alpha(|\alpha^{-1}(j) - \alpha^{-1}(i)|) \\ &= |j\dot{-}i|_N \end{aligned} \tag{3.1}$$

This shows that non-Newtonian distance is commutative.

Proposition 3.2 Let (M, d_N) be a non-Newtonian metric space and let $i,j,k,l \in M$. Then

$$|d_N(i,j)\dot{-}d_N(k,l)|_N \dot{\leq} d_N(i,k)\dot{+}d_N(j,l) \tag{3.2}$$

Proof The triangle inequality with the NNM axioms yields first

$$\begin{aligned} d_N(i,j) &\dot{\le} d_N(j,k) \dot{+} d_N(k,j) \\ &\dot{\le} d_N(j,k) \dot{+} d_N(k,l) \dot{+} d_N(l,j) \end{aligned}$$

Using the symmetry axiom, rearrangement of the above inequality gives

$$d_N(i,j) \dot{-} d_N(k,l) \dot{\le} d_N(i,k) \dot{+} d_N(j,l) \tag{3.3}$$

Similarly, we have

$$\begin{aligned} d_N(k,l) &\dot{\le} d_N(k,i) \dot{+} d_N(i,l) \\ &\dot{\le} d_N(k,i) \dot{+} d_N(i,j) \dot{+} d_N(j,l) \\ &= d_N(i,k) \dot{+} d_N(i,j) \dot{+} d_N(j,l) \end{aligned}$$

Therefore

$$d_N(k,l) \dot{-} d_N(i,k) \dot{\le} d_N(i,k) \dot{+} d_N(j,l) \tag{3.4}$$

Thus from (3.3) and (3.4) it follows that (3.2).

Proposition 3.3 Let $(M, \| \,. \|_N)$ be a non-Newtonian normed space. Then

$$| \|i\|_N \dot{-} \|j\|_N |_N \dot{\le} \|i \dot{-} j\|_N, \forall i,j \in M \tag{3.5}$$

Proof Observe that

$$\|i\|_N = \|i \dot{-} j \dot{+} j\|_N \dot{\le} \|i \dot{-} j\|_N \dot{+} \|j\|_N$$

Therefore $\|i\|_N \dot{-} \|j\|_N \dot{\le} \|i \dot{-} j\|_N$. Swapping the role of i and j, we also obtain $\|j\|_N \dot{-} \|i\|_N \dot{\le} \|i \dot{-} j\|_N$.This implies (3.5).

Now, we introduce some definitions in non-Newtonian metric spaces.

Definition 3.4 Suppose (M, d_N) is a non-Newtonian complete metric space. A mapping $f: M \to M$ is called non-Newtonian expansive if there exists a non-Newtonian number $\delta \dot{>} \dot{1}$ such that

$$d_N(fx, fy) \dot{\ge} \delta \dot{\times} d_N(x,y), \forall\, x,y \in M. \tag{3.6}$$

Definition 3.5 Let (M, d_N) be a non-Newtonian metric space and f be a self-mapping of M: (NN1) There exist non-Newtonian numbers a, b, c satisfying $b \dot{\ge} \dot{0}, c \dot{\ge} \dot{0}$ and $a \dot{>} \dot{1}$ such that

$$d_N\big(f(i), f(j)\big) \dot{\ge} a \dot{\times} d_N(i,j) \dot{+} b \dot{\times} d_N\big(i, f(i)\big) \dot{+} c \dot{\times} d_N\big(j, f(j)\big) \tag{3.7}$$

for each $i, j \in M$. In this case f is called non-Newtonian expansive type mapping.

Now, we give a simple but a useful Lemma.

Lemma 3.6 Let $\{j_n\}$ be a sequence in a non-Newtonian metric space such that

$$d_N(j_n, j_{n+1}) \le \delta \dot{\times} d_N(j_{n-1}, j_n) \tag{3.8}$$

where $\delta \dot{<} \dot{1}$ and $n \in \mathbb{N}$.Then $\{j_n\}$ is a non-Newtonian Cauchy sequence in M.

Proof By the simple induction with the condition (3.8), we have

$$\begin{aligned} d_N(j_n, j_{n+1}) &\dot{\leq} \delta \dot{\times} d_N(j_{n-1}, j_n) \\ &\dot{\leq} \delta^{2_N} \dot{\times} d_N(j_{n-2}, j_{n-1}) \\ &\dot{\leq} \delta^{n-1_N} \dot{\times} d_N(j_0, j_1) \end{aligned} \tag{3.9}$$

Now, if $m < n$, we have

$$\begin{aligned} d_N(j_n, j_m) &\dot{\leq} d_N(j_n, j_{n-1}) \dot{+} d_N(j_{n-1}, j_{n-2}) \dot{+} \ldots \dot{+} d_N(j_{m+1}, j_m) \\ &\dot{\leq} \delta^{n-1_N} \dot{\times} d_N(j_0, j_1) \dot{+} \delta^{n-2_N} \dot{\times} d_N(j_0, j_1) \dot{+} \ldots \dot{+} \delta^{m_N} \dot{\times} d_N(j_0, j_1) \\ &\dot{\leq} \delta^{m_N} \dot{\times} \left(\dot{1} \dot{+} \delta \dot{+} \delta^{2_N} \dot{+} \ldots \ldots \dot{+} \delta^{n-m-1_N}\right) \dot{\times} d_N(j_0, j_1) \\ &\dot{\leq} \frac{\delta^{m_N} \dot{\times} d_N(j_0, j_1)}{\dot{1} \dot{-} \delta} \end{aligned} \tag{3.10}$$

Since $\delta^{m_N} \dot{<} \dot{1}$ and $d_N(j_0, j_1) \in \mathbb{R}(N)$ is fixed, we can make $\frac{\delta^{m_N} \dot{\times} d_N(j_0, j_1)}{\dot{1} \dot{-} \delta}$ as small as we want by taking m sufficiently large. This shows that $\{j_n\}$ is a non-Newtonian Cauchy sequence.

Now, we give some fixed-point results for expansive mappings in a non-Newtonian complete metric space. Our first main result as follows.

Theorem 3.7 Let $f: M \to M$ be a surjection and non-Newtonian expansive mapping on a non-Newtonian complete metric space M. Then f has a unique fixed point.

Proof: Let $j_0 \in M$ be arbitrary. Since f is surjection, then there exists $j_1 \in M$ such that $j_0 = f(j_1)$. By continuing this process, we get

$$j_n = f(j_{n+1}),\ n = 0, 1, 2, \ldots.. \tag{3.11}$$

In case $j_{n_0} = j_{n_0+1}$ for some n_0, then it is clear that j_{n_0} is a fixed point of f. Now assume that $j_n \neq j_{n-1}$ for all n. Since f non-Newtonian expansive mapping

$$d_N(j_{n-1}, j_n) = d_N\big(f(j_n), f(j_{n+1})\big) \dot{\geq} \delta \dot{\times} d_N(j_n, j_{n+1})$$

Consequently

$$d_N(j_n, j_{n+1}) \dot{\leq} \left(\dot{1} \dot{/} \delta\right) \dot{\times} d_N(j_{n-1}, j_n) = \kappa \dot{\times} d_N(j_{n-1}, j_n) \tag{3.12}$$

where $\kappa = \dot{1} \dot{/} \delta \dot{<} \dot{1}$.

Then by Lemma 3.6, $\{j_n\}$ is an NN-Cauchy sequence. Since (M, d_N) is non-Newtonian complete, there exists a point j in M such that $j_n \xrightarrow{N} j$. Since f is surjection on M, there exists $u \in M$ such that $j = f(u)$. We now show that j is a fixed point of the mapping f. It follows from (3.6) and (3.11) that

$$d_N(j_n, j) = d_N\big(f(j_{n+1}), f(u)\big) \dot{\geq} \delta \dot{\times} d_N(j_{n+1}, u)$$

Since $j_n \xrightarrow{N} j$, it follows that $d_N(j_{n+1}, u) \xrightarrow{N} \dot{0}$ and hence $j_{n+1} \xrightarrow{N} u$. By uniqueness of non-Newtonian limit, we have $j = u$. This shows that j is a fixed point of f. We conclude the proof by showing that j is the only fixed point. Suppose that k is also a fixed point, that is, suppose $f(k) = k$, then

$$d_N(j, k) = d_N\big(f(j), f(k)\big) \dot{\geq} \delta \dot{\times} d_N(j, k)$$

Since $\delta \dot{>} \dot{1}$, this implies that $d_N(j, k) = \dot{0}$ and hence $j = k$.

Theorem 3.8 Let (M, d_N) be a non-Newtonian complete metric space and let f be a surjective self-mapping of M. If f satisfies condition $(NN1)$, then f has a unique fixed point in M.

Proof. Using the hypothesis, it can be easily seen that f is injective. Indeed, if we take $f(i) = f(j)$, then, using (3.7), we get

$$\dot{0} = d_N\big(f(i), f(j)\big) \dot{\geq} a \dot{\times} d_N(i, j) \dot{+} b \dot{\times} d_N\big(i, f(i)\big) \dot{+} c \dot{\times} d_N\big(j, f(j)\big)$$

And so $d_N(i, j) = \dot{0}$; that is, we have $i = j$, since $a \dot{>} \dot{1}$.

Let us denote the inverse mapping of f by F. Let $j_0 \in M$ and define the sequence $\{j_n\}$ as follows:

$$\begin{aligned} &j_1 = F(j_0), \quad j_2 = F(j_1) = F^2(j_0), \\ &j_3 = F(j_2) = FF^2(j_0) = F^3(j_0), \ldots\ldots, j_{n+1} = F(j_n) = F^{n+1}(j_0), \end{aligned} \tag{3.13}$$

Suppose that $j_n \neq j_{n+1}$ for all n. Using (3.7) and (3.13), we have

$$\begin{aligned} d_N(j_{n-1}, j_n) &= d_N\big(ff^{-1}(j_{n-1}), ff^{-1}(j_n)\big) \\ &\dot{\geq} a \dot{\times} d_N\big(f^{-1}(j_{n-1}), f^{-1}(j_n)\big) \dot{+} b \dot{\times} d_N\big(f^{-1}(j_{n-1}), ff^{-1}(j_{n-1})\big) \\ &\qquad \dot{+} c \dot{\times} d_N\big(f^{-1}(j_n), ff^{-1}(j_n)\big) \\ &\dot{\geq} a \dot{\times} d_N\big(F(j_{n-1}), F(j_n)\big) \dot{+} b \dot{\times} d_N(F(j_{n-1}), j_{n-1}) \dot{+} c \dot{\times} d_N(F(j_n), j_n) \\ &\dot{\geq} a \dot{\times} d_N(j_n, j_{n+1}) \dot{+} b \dot{\times} d_N(j_n, j_{n-1}) \dot{+} c \dot{\times} d_N(j_{n+1}, j_n) \\ &= (a \dot{+} c) \dot{\times} d_N(j_n, j_{n+1}) \dot{+} b \dot{\times} d_N(j_n, j_{n-1}) \end{aligned}$$

which implies that

$$\big(\dot{1} \dot{-} b\big) \dot{\times} d_N(j_{n-1}, j_n) \dot{\geq} (a \dot{+} c) \dot{\times} d_N(j_n, j_{n+1}) \tag{3.14}$$

Clearly, we have $a \dot{+} c \neq \dot{0}$. Hence, we obtain

$$d_N(j_n, j_{n+1}) \dot{\leq} \big(\dot{1} \dot{-} b\big) \dot{/} (a \dot{+} c) \dot{\times} d_N(j_{n-1}, j_n) = \delta \dot{\times} d_N(j_{n-1}, j_n) \tag{3.15}$$

Where $\delta = \big(\dot{1} \dot{-} b\big) \dot{/} (a \dot{+} c)$, then we get $\delta \dot{<} \dot{1}$, since $a \dot{+} b \dot{+} c \dot{>} 1$. Repeating this process in condition (3.15), we find

$$d_N(j_n, j_{n+1}) \dot{\leq} \delta^{n_N} \dot{\times} d_N(j_0, j_1)$$

and by Lemma 3.6, $\{j_n\}$ is an NN-Cauchy sequence. Since (M, d_N) is non-Newtonian complete, there exists a point j in M such that $j_n \xrightarrow{N} j$ and therefore

$$d_N(j_n, j) \xrightarrow{N} \dot{0}, \quad d_N(j_{n+1}, j_n) \xrightarrow{N} \dot{0}.$$

Using the subjectivity of hypothesis, there exists $u \in M$ such that $j = f(u)$. From (3.7) and (3.13), we have

$$\begin{aligned} d_N(j_n, j) &= d_N\big(f(j_{n+1}), f(u)\big) \\ &\dot{\geq} a \dot{\times} d_N(j_{n+1}, p) \dot{+} b \dot{\times} d_N\big(j_{n+1}, f(j_{n+1})\big) \dot{+} c \dot{\times} d_N\big(u, f(u)\big) \\ &= a \dot{\times} d_N(j_{n+1}, p) \dot{+} b \dot{\times} d_N(j_{n+1}, j_n) \dot{+} c \dot{\times} d_N\big(u, f(u)\big) \end{aligned}$$

If we take limit for $n \xrightarrow{N} \infty$, we obtain

$$\dot{0} \dot{\geq} (a \dot{+} c) \dot{\times} d_N(u, j)$$

which implies that $d_N(u,j) = \dot{0}$; that is, we have $j = u$, since $a \dot{+} c \dot{>} \dot{1}$. This shows that j is a fixed point of f.

Now we show the uniqueness of j. Let k be another fixed point of f with $j \neq k$. Using (3.7), we get

$$\begin{aligned} d_N(j,k) &= d_N\big(f(j), f(k)\big) \\ &\dot{\geq} a \dot{\times} d_N(j,k) \dot{+} b \dot{\times} d_N\big(j, f(j)\big) \dot{+} c \dot{\times} d_N\big(k, f(k)\big) \\ &= a \dot{\times} d_N(j,k) \dot{+} b \dot{\times} d_N(j,j) \dot{+} c \dot{\times} d_N(k,k) \\ &= a \dot{\times} d_N(j,k) \end{aligned} \tag{3.16}$$

which implies that $j = k$, since $a \dot{>} \dot{1}$. Consequently, f has a unique fixed point j.

If we take $b = c$ in condition $(NN1)$, then we obtain the following corollary.

Corollary 3.9 Let (M, d_N) be a non-Newtonian complete metric space and let f be a surjective self-mapping of M. If there exist real numbers a, b satisfying $b \dot{\geq} \dot{0}$ and $a \dot{>} \dot{1}$ such that

$$d_N\big(f(i), f(j)\big) \dot{\geq} a \dot{\times} d_N(i,j) \dot{+} b \dot{\times} max\{d_N\big(i, f(i)\big), d_N\big(j, f(j)\big)\} \tag{3.17}$$

for each $i, j \in M$, then f has a unique fixed point in M.

Now, we prove following common fixed point result.

Theorem 3.10 Let $f, g: M \to M$ be two surjective mappings of a non-Newtonian complete metric space (M, d_N). Suppose that f and g satisfying inequalities

$$d_N\Big(f\big(g(j)\big), g(j)\Big) \dot{+} \kappa \dot{\times} d_N\big(f\big(g(j)\big), j\big) \dot{\geq} a \dot{\times} d_N(g(j), j) \tag{3.18}$$

$$d_N\Big(g\big(f(j)\big), f(j)\Big) \dot{+} \kappa \dot{\times} d_N\big(g\big(f(j)\big), j\big) \dot{\geq} b \dot{\times} d_N(f(j), j) \tag{3.19}$$

for $j \in M$ and some non-Newtonian real numbers a, b and κ with $a \dot{-} \kappa \dot{>} \dot{1} \dot{+} k$ and $b \dot{-} \kappa \dot{>} \dot{1} \dot{+} \kappa$. If f or g is non-Newtonian continuous, then f and g have a common fixed point in M.

Proof Let j_0 be an arbitrary point in M. Since f is surjective, there exists $j_1 \in M$ such that $j_0 = f(j_1)$. Also, since g is surjective, there exists $j_2 \in M$ such that $j_2 = g(j_1)$. Continuing this process, we construct a sequence $\{j_n\}$ in M such that $j_{2n} = f(j_{2n+1})$ and $j_{2n+1} = g(j_{2n+2})$ for all $n \in \mathbb{N}$. Now for $n \in \mathbb{N}$, by (3.18) we have

$$d_N\Big(f\big(g(j_{2n+2})\big), g(j_{2n+2})\Big) \dot{+} \kappa \dot{\times} d_N\big(f\big(g(j_{2n+2})\big), j_{2n+2}\big) \dot{\geq} a \dot{\times} d_N(g(j_{2n+2}), j_{2n+2})$$

Thus

$$d_N(j_{2n}, j_{2n+1}) \dot{+} \kappa \dot{\times} d_N(j_{2n}, j_{2n+2}) \dot{\geq} a \dot{\times} d_N(j_{2n+1}, j_{2n+2})$$

which implies that

$$d_N(j_{2n}, j_{2n+1}) \dot{+} \kappa \dot{\times} [d_N(j_{2n}, j_{2n+1}) \dot{+} d_N(j_{2n+1}, j_{2n+2})] \dot{\geq} a \dot{\times} d_N(j_{2n+1}, j_{2n+2})$$

Hence

$$d_N(j_{2n+1}, j_{2n+2}) \dot{\leq} \big[\big(\dot{1} \dot{+} \kappa\big) \dot{/} (a \dot{-} \kappa)\big] \dot{\times} d_N(j_{2n}, j_{2n+1}) \tag{3.20}$$

On other hand, from (3.19), we have

$$d_N\left(g\big(f(j_{2n+1})\big), f(j_{2n+1})\right) \dot{+} \kappa \dot{\times} d_N\big(g\big(f(j_{2n+1})\big), j_{2n+1}\big) \dot{\geq} b \dot{\times} d_N(f(j_{2n+1}), j_{2n+1})$$

Thus

$$d_N(j_{2n-1}, j_{2n}) \dot{+} \kappa \dot{\times} d_N(j_{2n-1}, j_{2n+1}) \dot{\geq} b \dot{\times} d_N(j_{2n}, j_{2n+1})$$

which implies that

$$d_N(j_{2n-1}, j_{2n}) \dot{+} \kappa \dot{\times} [d_N(j_{2n-1}, j_{2n}) \dot{+} d_N(j_{2n}, j_{2n+1})] \dot{\geq} b \dot{\times} d_N(j_{2n}, j_{2n+1})$$

Hence

$$d_N(j_{2n}, j_{2n+1}) \dot{\leq} \left[\big(\dot{1} \dot{+} \kappa\big) \dot{/} (b \dot{-} \kappa)\right] \dot{\times} d_N(j_{2n-1}, j_{2n}) \tag{3.21}$$

Let $\delta = maj\{[(\dot{1} \dot{+} \kappa) \dot{/} (a \dot{-} \kappa)], [(\dot{1} \dot{+} v) \dot{/} (b \dot{-} \kappa)]\} \dot{<} \dot{1}$

Then by combining (3.20) and (3.21), we have

$$d_N(j_n, j_{n+1}) \leq \delta \dot{\times} d_N(j_{n-1}, j_n) \tag{3.22}$$

where $\delta \in \dot{[}0,1\dot{)}$, $\forall n \in \mathbb{N}$. Then by Lemma 3.6, the sequence $\{j_n\}$ is an NN-Cauchy sequence. Since (M, d_N) is non-Newtonian complete, there exists a point j in M such that $j_n \xrightarrow{N} j$. Therefore $j_{2n+1} \xrightarrow{N} j$ and $j_{2n+2} \xrightarrow{N} j$ as $n \to +\infty$. Without loss of generality, we may assume that f is continuous, then $f(j_{2n+1}) \xrightarrow{N} f(j)$ as $n \to +\infty$. But $f(j_{2n+1}) = j_{2n} \xrightarrow{N} j$ as $n \to +\infty$. Thus, we have $f(j) = j$. Since g is surjection on M, there exists $u \in M$ such that $j = g(u)$. We now show that j is a common fixed point of the mapping f and g. It follows from (3.18) that

$$\begin{aligned} & d_N\left(f\big(g(u)\big), g(u)\right) \dot{+} \kappa \dot{\times} d_N\big(f\big(g(u)\big), u\big) \dot{\geq} a \dot{\times} d_N(g(u), u) \\ \Rightarrow \quad & \dot{0} \dot{+} \kappa \dot{\times} d_N(j, u) \dot{\geq} a \dot{\times} d_N(j, u) \\ \Rightarrow \quad & \dot{0} \dot{\geq} (a \dot{-} \kappa) \dot{\times} d_N(j, u) \end{aligned}$$

Since $a \dot{-} \kappa \dot{>} \dot{1} \dot{+} \kappa$, we conclude that $d_N(j, u) = \dot{0}$ and consequently $j = u$. Hence $f(j) = g(j) = j$. Therefore j is a common fixed point of f and g.

By taking $f = g$ in Corollary 3.9 we have the following Corollary.

Corollary 3.10 Let $f: M \to M$ be two surjective mappings of a non-Newtonian complete metric space (M, d_N). Suppose that f satisfying inequality

$$d_N\big(f^2(j), f(j)\big) \dot{+} \kappa \dot{\times} d_N(f^2(j), j) \dot{\geq} a \dot{\times} d_N(f(j), j) \tag{3.24}$$

for $j \in M$ and some nonnegative real numbers a, b and k with $a \dot{-} \kappa \dot{>} \dot{1} \dot{+} \kappa$. If f is continuous, then f has a fixed point in M.

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Competing Interests

Authors have declared that no competing interests exist.

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