Asian Research Journal of Mathematics

2(3): 1-13, 2017; Article no.ARJOM.31195

ISSN: 2456-477X

SCIENCEDOMAIN *international*

www.sciencedomain.org

Modeling the Traffic Accident Data Using a Convenient Lognormal Diffusion Process

Basel M. Al-Eideh[1*]

[1]*Department of Quantitative Methods and Information Systems, Kuwait University - Collage of Business Administration, P.O.Box 5486, Safat 13055, Kuwait.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/31195

Editor(s):

(1) Andrej V. Plotnikov, Department of Applied and Calculus Mathematics and CAD, Odessa State Academy of Civil Engineering and Architecture, Ukraine.

Reviewers:

(1) Matheus Henrique Dal Molin Ribeiro, Federal Technological University of Paraná, Brazil.

(2) Petras Rupsys, Aleksandras Stulginskis University, Lithuania.

(3) Suchandan Kayal, National Institute of Technology Rourkela, India.

(4) Radosław Jedynak, Kazimierz Pulaski University of Technology and Humanities, Radom, Poland.

Complete Peer review History: http://www.sciencedomain.org/review-history/17707

Received: 26th December 2016

Accepted: 24th January 2017

Published: 3rd February 2017

Original Research Article

Abstract

Aims: Theories of diffusion process play an important role in safety traffic applications. The purpose of this paper is to introduce a methodology capable for fitting the yearly traffic accidents in Kuwait. More specifically, a lognormal diffusion model is specified to be capable to fit and explicitly addressing the variations in the yearly traffic accidents in Kuwait during the period 2002-2013. Estimation results using the maximum likelihood estimation and the goodness of fit using the sample autocorrelation function clearly demonstrated the appropriateness of the estimated lognormal diffusion model.

Study Design: A rigorous and mathematically sound model is specified to study the yearly total number of traffic accidents in Kuwait.

Place and Duration of Study: Department of Quantitative Methods and Information Systems, College of Business Administration, Kuwait University, Kuwait, during the year 2016.

Methodology: A lognormal diffusion process is considered and estimation is done using the MLE method and goodness of fit using sample ACF and partial ACF are also used to prove the capability and the appropriateness of the suggested lognormal diffusion model to fit the yearly traffic accidents data in Kuwait during the period 2002-2013.

Results: To estimate the previously specified lognormal diffusion model, we use the yearly observations of the total number of accident data in Kuwait from 2002 to 2013 were obtained from the Kuwait Traffic

*Corresponding author: E-mail: basel@cba.edu.kw;

Police Department. The maximum likelihood estimates of model parameters are given by $\hat{\beta}$ = 0.0342, $\hat{\delta}$ = 0.00125, $\hat{a}$ = 0.00125 and $\hat{b}$ = 0.03482. Using the fitted model, we predict the yearly number of accidents for the period from 2014 to 2020. The capability of the model is supported by the residual analysis through the autocorrelation and the normal probability plots.
Conclusion: This study provided a methodology capable and convenient to fit the traffic accident data and departs from the traditional before-and-after regression techniques and the time series analysis and developed a diffusion model that explicitly accounts for the variations in the total number of accidents.

Keywords: Lognormal diffusion process; traffic accidents; maximum likelihood estimation.

1 Introduction

Decision makers usually ask the question whether the implementation of speed limits, seat belt laws, or sudden increase in population due to random event(s) have an impact on the extent and security of traffic-related accidents. To give a satisfactory response, a rigorous and theoretically sound approach ought to be used.

It has reported in the literature that the choice of a statistical technique is likely to affect the impact of an event. Rock [1] addressed the effect of the 1985 seat belt-use law in Illinois on risk compensation. Tow statistical techniques were used in the study. The first relates to a before-and-after method and the second relates to intervention analysis using ARIMA techniques. Unlike the before-and-after method, the developed time-series intervention models showed no statistically significant increase in traffic accidents. However, conclusions reported in the study contradicted those reported by Asch et al. [2]. Evans and Graham [3], and Garbacz [4,5], who used the traditional before-and-after analysis, Rock [1] argued that the presence of correlated error terms and model misspecification plays against the use of regression methods.

Other studies in the literatures have focused on the impact of the 55-mph speed limit on traffic accidents. During the 1973 energy crisis, a national speed limit of 55-mph was imposed in the USA. Since then, a number of studies have evaluated the impact of such speed limit on traffic accidents (TRB, [6]). Helfenstein [7] addressed the effect of reduced speed limit in the propensity of traffic accident occurrence. The study dealt specifically with the time (time intervention) when the reduced speed limit had a significant effect on the magnitude of traffic accidents.

Hamed et al. [8] specified a rigorous and mathematically sound model for studying the influence of the recent Gulf crisis (2 August 1990) on the monthly total number of traffic accidents in Jordan. A stochastic diffusion model was specified and estimated in the study. They found that the Gulf crisis did not have a significant impact on the total number of traffic accidents.

Concerning modeling of the traffic accidents, there are series of studies available in the literature and are discussed from different points of views of their authors, such as Caliendo et al. [9], Park and Lord [10], Ma et al. [11], Cafiso et al. [12], Chen and Zhang [13], Lord and Mannering [14], Malyshkina and Mannering [15], Anastasopoulos and Mannering [16], Jovanovic et al. [17], Savolainen et al. [18], Wang, et al. [19], Castro et al. [20], Geedipally et al. [21], Aguero-Valverde [22], Bhat et al. [23], Caliendo et al. [24], Chiou and Fu [25], Deublein et al. [26], Narayanamoorthy et al. [27], Zou et al. [28].

More specifically, Two relatively recently published papers provide a comprehensive review of current methodological approaches for studying crash frequencies, the number of crashes on a roadway segment or intersection over some specified time period (Lord and Mannering, [14]), and crash severities, usually measured by the most severely injured person involved in the crash (Savolainen et al. [18]).

Castro et al. [20], Narayanamurthi et al. [27] and Bhat et al. [23] have recast count models as a restrictive case of a generalized ordered-response model, with a latent long-term risk propensity for crashes coupled with thresholds that determine the translation of that risk to the instantaneous probability of a crash outcome. Such a generalized ordered-response approach to count data has several potential advantages; including making it much easier to extend univariate count models to multivariate count models and accommodating spatial and temporal dynamics.

Wang, et al. [19] proposed an alternative method to estimate accident frequency at different severity levels, namely the two-stage mixed multivariate model which combines both accident frequency and severity models. The accident, traffic and road characteristics data from the M25 motorway and surrounding major roads in England have been collected to demonstrate the use of the two-stage model. They found that the two-stage mixed multivariate model is a promising tool in predicting accident frequency according to their severity levels and site ranking.

Although intervention analyses account for the variation in traffic accident data, they are unable to explain the behavior of the traffic accident process. Time-series models are normally used to reproduce observed data (provide a goodness of fit to observed data) through a somewhat subjective mechanism.

In this paper, a rigorous and mathematically sound model is specified to study the yearly total number of traffic accidents in Kuwait. A lognormal diffusion process is considered and estimation is done using the MLE method and goodness of fit using sample ACF and partial ACF are also used to prove the capability and the appropriateness of the suggested lognormal diffusion model to fit the yearly traffic accidents data in Kuwait during the period 2002-2013.

2 Methodology

Diffusion processes have been used in the literature to model many physical, biological, economics and social problems. Examples include molecular motions of enumerable particles subject to interactions, security price fluctuations in a perfect market, neurophysiological activity with disturbances, variations in population growth, changes in number of species subject to competitions, and gene distributions in evolutionary development (Brockwell et al. [29]; Brockwell, [30]; Sorensen, [31]).

The lognormal diffusion process has been widely used in stochastic modeling in different scientific fields. For example, Rrez et al. [32] presented a methodology to build a lognormal discussion process with exogenous factors that models economic variables to the GNP of Spain. This model allows them to study the problems of forecasting as first-passage-times.

Tintner and Patel [33] tried to apply a lognormal diffusion process to the development of the Indian economy. The use of the lognormal rather than the normal or the Pareto distributions is suggested by a number of investigations which seem to point to its usefulness as a convenient approximation in a number of fields such as economics (cf. Sprekle [34] and Granger and Morgenstern [35]).

The intent of this paper is not to replicate the detailed discussions of the methodological alternatives provided in those papers, but instead to focus on using the lognormal diffusion process to fit the yearly traffic accidents.

Assume $X(t)$ be the yearly traffic accidents at time t, then $\{X(t); t \geq 0\}$ be a continuous Markov process defined on the state space interval $[0,\infty)$ of real line, note that $X(t)$ is a Markovian random variable depending on a continuous time parameter t, which assumes values in the state space $S=[0,\infty)$.

Let the transition probability density of $X(t)$ is given by

$$f(\tau, x : t, y) = \Pr(X(t) = y; X(\tau) = x) \; ; \;\; 0 < y, x < \infty \tag{1}$$

exists for every τ and t, where $0 \le \tau \le t$, and satisfies the backward and forward Kolmogorov equations. Also, assume the random variable $X(t)$ is continuous with probability one.

Assume $X(t)$ represent the diffusion process by which total number of accidents at time t is accumulated with the infinitesimal mean $\mu(t,x)$ and infinitesimal variance $\sigma(t,x)$ of the change in $X(t)$ during a small interval Δt of time, i.e.

$$\mu(t,x) = b_t x = bx \tag{2}$$

and

$$\sigma(t,x) = a_t x^2 = ax^2 \tag{3}$$

We assume $b_t = b$ and $a_t = a$, where b and a are constants and are called the drift and the diffusion parameters where $a > 0$. These assumptions yield the lognormal diffusion process (cf. Tintner and Patel, [33]).

This assumes that the expected change and its variance in the total number of accidents are proportional to the instantaneous size of it. And with $a > 0$, we mean that some change in the total number of accidents will take place in any interval Δt and it will be small if Δt is small. This seems to well describe the plausible characteristics of a traffic data. Note that the parameter b represents the net increase in the total number of accidents.

Thus the backward and forward Kolmogorov equations are respectively:

$$\frac{\partial f}{\partial \tau} = \frac{1}{2} a x^2 \frac{\partial^2 f}{\partial x^2} - bx \frac{\partial f}{\partial x} \tag{4}$$

and

$$\frac{\partial f}{\partial t} = \frac{1}{2} a y^2 \frac{\partial^2 f}{\partial y^2} + (2a-b) y \frac{\partial f}{\partial y} + (a-b) f \tag{5}$$

The probability density function satisfying these equations (4) and (5) is then given by the lognormal density function.

$$f_\theta(\tau,x,t,y) = \frac{1}{y[2\pi\delta(t-\tau)]^{1/2}} e^{\left\{\left(-\frac{1}{2\delta(t-\tau)}\right)(\log y - \log x - \beta(t-\tau))^2\right\}} \tag{6}$$

where $\delta = a$ and $\beta = \left(b - \frac{a}{2}\right)$; $\theta = (\delta, \beta)$.

The characteristics of this distribution are easily derived from its moments. Thus the k-th moment of $X(t)$ is then given by

$$E[X(t)]^k = \left(X(\tau)\right)^k \cdot e^{\left(k\beta + k^k \cdot \frac{\delta}{2}\right)(t-\tau)}, \qquad k = 1,2,3,\cdots.$$

which finally gives the mean $\mu(X(t))$ and the variance $Var(X(t))$ as follows

$$\mu(X(t)) = E[X(t)] = \left(X(\tau)\right) \cdot e^{\left(\beta + \frac{\delta}{2}\right)(t-\tau)}$$
$$= X(\tau) \cdot e^{b(t-\tau)}$$

$$Var(X(t)) = \left(X(t)\right)^2 \cdot e^{2b(t-\tau)} \cdot \left(e^{a(t-\tau)} - 1\right)$$

Note that when $\tau = 0$, the mean and variance are exponential functions of t.

Also, lognormal distribution is very attractive distribution for several reasons such as it is very well known and understood by many people that mean that results can be easily explained. It is considerably easier to manipulate mathematically. Finally, the model is a lot easier implement, so saving time and money.

3 Case Study

To estimate the previously specified lognormal diffusion model, we use the yearly observations of the total number of accident data in Kuwait from 2002 to 2013 were obtained from the Kuwait Traffic Police Department. Fig. 1 shows the distribution of these observations.

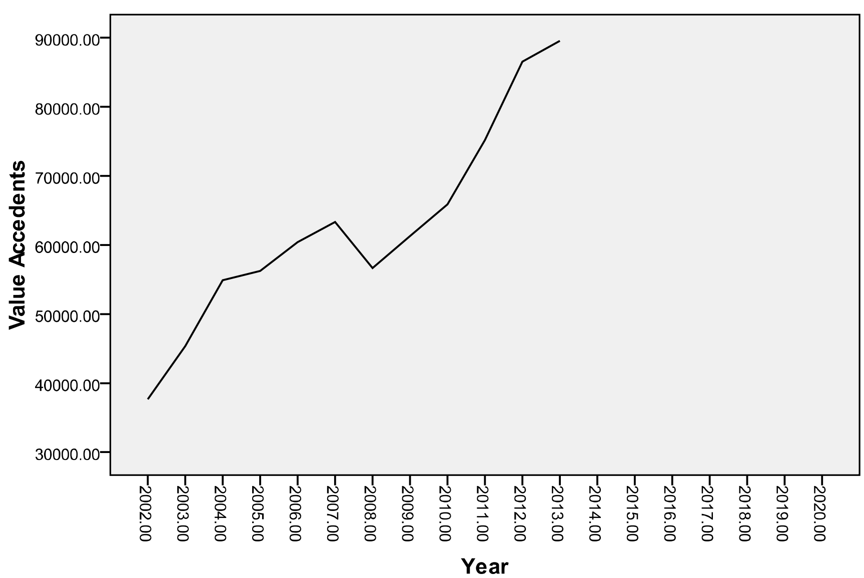

Fig. 1. The real traffic accidents

The method of maximum likelihood estimation (MLE) is used to estimate the model parameters b and a. Now it is sufficient to find MLE of the parameters δ and β and then by the invariance property of the MLE, we find the MLE's of b and a.

Let $X(0) = x_0$, $X(1) = x_1$, $X(2) = x_2$, ... , $X(n) = x_n$ represent observations of the yearly total number of accidents at times 0,1,2,3,…,n. Conditioning on $X(0) = x_0$, the likelihood function can be written as (cf. Brockwell [30] and Sorensen [31]).

$$\mathrm{L}(x_1, x_2 \ldots. x_n / x_0) = \prod_{i=0}^{n-1} f_\theta(x_i ; x_{i+1}) \tag{7}$$

where $f_\theta(x_i ; x_{i+1})$ for *i=0,1,2…,n-1* is the one-step transition probability density from state x_i to x_{i+1} and $\boldsymbol{\theta}$ is a vector of estimated parameters (δ and β). (Note that $t_{i+1} - t_i = 1$; *i*= *0,1,2…,n-1* and $t_0 = 0$).

Thus, L ($x_1, \ldots, x_n / x_0$) can be written as

$$\mathrm{L} = \left(\frac{1}{(2\pi\delta)^{1/2}}\right)^n \left(\prod_{i=1}^{n} \frac{1}{x_i}\right) . e^{\left\{-\frac{1}{2\delta}\sum_{i=1}^{n} [\log x_i - \log x_{i-1} - \beta]^2\right\}} \tag{8}$$

The maximum likelihood estimates are given by solving the equations.

$$\frac{\partial \log L}{\partial \hat{\beta}} = \left(\frac{1}{\hat{\delta}}\right) \sum_{i=1}^{n} \left[\log x_i - \log x_{i-1} - \hat{\beta}\right] = 0 \tag{9}$$

and

$$\frac{\partial \log L}{\partial \hat{\delta}} = -\frac{n}{2\hat{\delta}} + \frac{1}{2\hat{\delta}^2} \sum_{i=1}^{n} \left[\left(\log x_i - \log x_{i-1} - \hat{\beta}\right)^2\right] = 0 \tag{10}$$

Solving equations (9) and (10) simultaneously, we get the estimates as follows:

$$\hat{\beta} = \sum_{i=1}^{n} \frac{(\log x_i - \log x_{i-1})}{n} \tag{11}$$

and

$$\hat{\delta} = \sum_{i=1}^{n} \frac{(\log x_i - \log x_{i-1})^2}{n} - \hat{\beta}^2 \tag{12}$$

Hence the MLE of a and b are as follows:

$$\hat{a} = \hat{\delta} \tag{13}$$

and

$$\hat{b} = \hat{\beta} + \frac{\hat{\delta}}{2} \tag{14}$$

Also, it is easily shown that the variances of our estimates $\hat{a}$ and $\hat{b}$, i.e.

$$Var(\hat{a}) = \frac{2\delta}{n} \qquad and \qquad Var(\hat{b}) = \frac{\delta^2}{2n} + \frac{\delta}{n}\cdot\frac{1}{t} \rightarrow \frac{\delta^2}{2n} \qquad as \qquad n \rightarrow \infty.$$

And covariance of $(\hat{a},\hat{b})$ is given by $Cov(\hat{a},\hat{b}) = 0$. Note that for large number of observations the variances above also be minimized. Thus the above estimates and their sampling variances and those nice properties shows that the MLE's are efficient and consistence as well as the invariance property made the maximum likelihood estimates is our method of choice to fit these kind of data of the real total number of accidents of Kuwait State.

4 Estimation Results

We now fit the lognormal diffusion model to the data of the yearly real total number of accidents of the Kuwait State during the period 2002 to 2013. Using the Best-Fit program, the maximum likelihood estimates of β and δ are given by

$$\hat{\beta} = 0.0342 \qquad \text{and} \qquad \hat{\delta} = 0.00125$$

and then using equations (13) and (14) we get :

$$\hat{a} = 0.00125 \qquad \text{and} \qquad \hat{b} = 0.03482$$

with standard deviation of the estimates $\hat{a}$ and $\hat{b}$ of 0.01443 and 0.00036, respectively.

The fitted model of the Kuwait yearly total number of accidents $X(t)$ is defined to be the expected value of the yearly total number of accident value at time t given that the value of yearly total number of accident at time $t-1$ has already observed, i.e. $E[X(t)/X(t-1)]$. It is easily shown that

$$E[X(t)/X(t-1)] = X(t-1)e^{\left\{\hat{b}(t_i - t_{i-1})\right\}} \tag{15}$$

For simplicity, we use $E[X(t)]$ to denote $E[X(t)/X(t-1)]$, and since $t_i - t_{i-1} = 1$, we get the fitted model for the yearly total number of accidents data as follows

$$E[X(t)] = X(t-1)e^{\hat{b}} \tag{16}$$

Now, Fig. 2 below shows the real and the predicted yearly number of accidents in Kuwait during the period 2002 to 2013. The figure shows that the lognormal diffusion model resembles the yearly traffic accident data. Also, shows the predicted yearly number of accidents for the period from 2014 to 2020. See also, Table 1 below for real and the predicted yearly traffic accidents data in Kuwait.

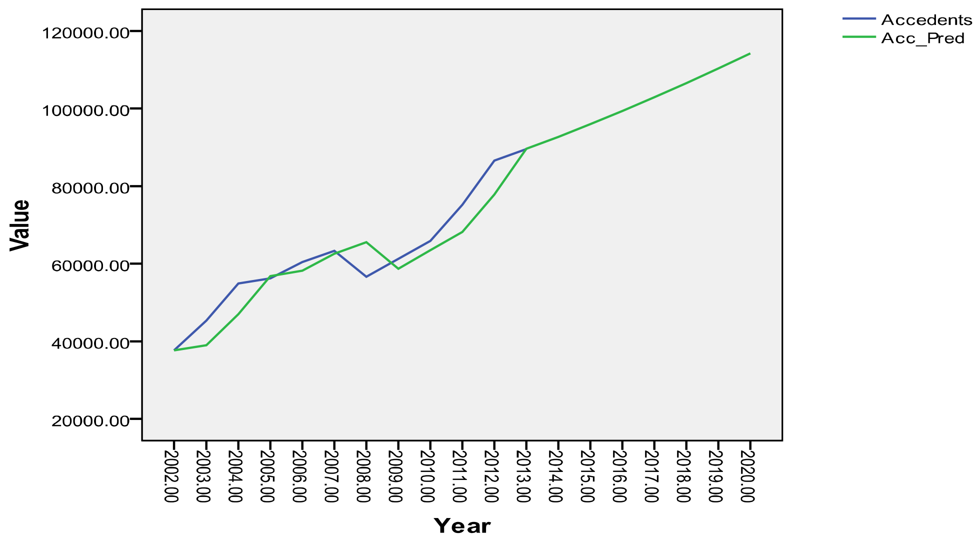

Fig. 2. The real and the predicted traffic accidents

Table 1. The real and the predicted yearly traffic accidents in Kuwait

Year	Real traffic accidents	Predicted traffic accidents
2002	37650	37650
2003	45376	38984
2004	54878	46984
2005	56235	56823
2006	60410	58228
2007	63323	62551
2008	56660	65567
2009	61298	58668
2010	65861	63470
2011	75194	68195
2012	86542	77859
2013	89527	89609
2014		92700
2015		95985
2016		99386
2017		102908
2018		106555
2019		110331
2020		114241

5 Residual Analysis

The goodness-of-fit of the developed lognormal diffusion model is checked using a test based on the sample autocorrelation function (ACF) and the sample partial ACF as well as the normal probability plot of the scaled residuals ($W(n)$).

Let $R(n)$ be the residual, i.e.

$$R(n) = X(n) - E[X(n)] \tag{17}$$

Let μ be the mean of the residuals $R(n)$ and σ is the standard deviation of the residuals $R(n)$, (note that in our case $\hat{\mu} = 2579$ and $\hat{\sigma} = 5020.75$). Thus the scaled residuals are then defined by:

$$W(n) = \frac{R(n) - \hat{\mu}}{\hat{\sigma}} \tag{18}$$

The structure of the scaled residuals ($W(n)$) is used as a tool to determine the goodness-of-fit of the estimated lognormal diffusion model. More specifically, the residuals should be independently distributed with mean zero and constant variance. Plotting the sample autocorrelation function of ($W(n)$) and comparing these values with the bounds ($\pm 1.96/\sqrt{n}$) checks the compatibility with independence. If more than 5% of the values lie outside the bounds then residual's independence is rejected. Fig. 3 shows the sample autocorrelation function of $W(n)$. The figure clearly shows that the model's residuals pass the independence test, thus lending support to the overall goodness-of-fit of the estimated lognormal diffusion model.

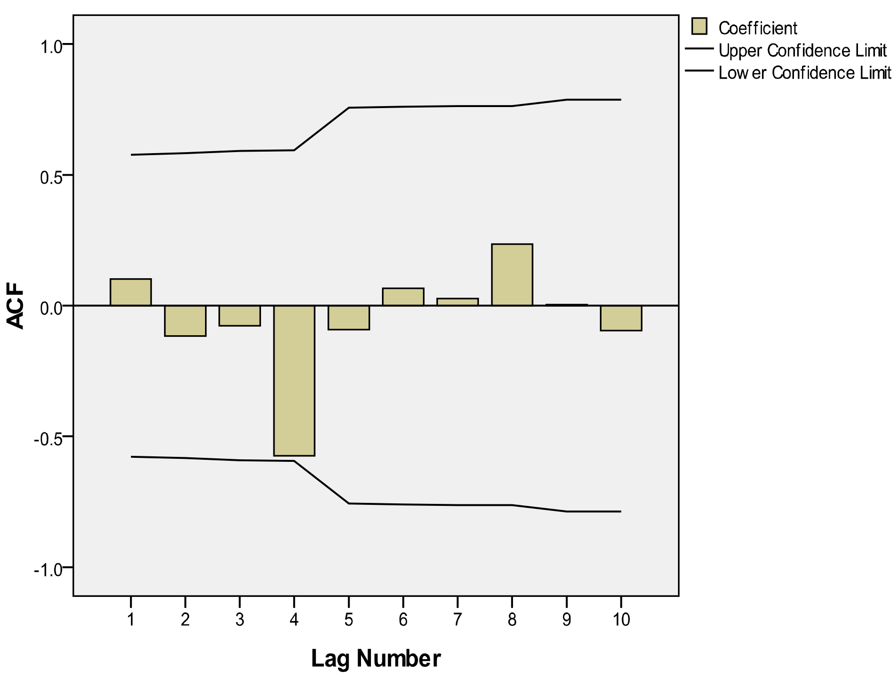

Fig. 3. The sample ACF of scaled residuals

Also, the sample partial auto correlation in Fig. 4 and the normal probability plot in Fig. 5 of the scaled residuals $W(n)$ below support the compatibility of the fitted model.

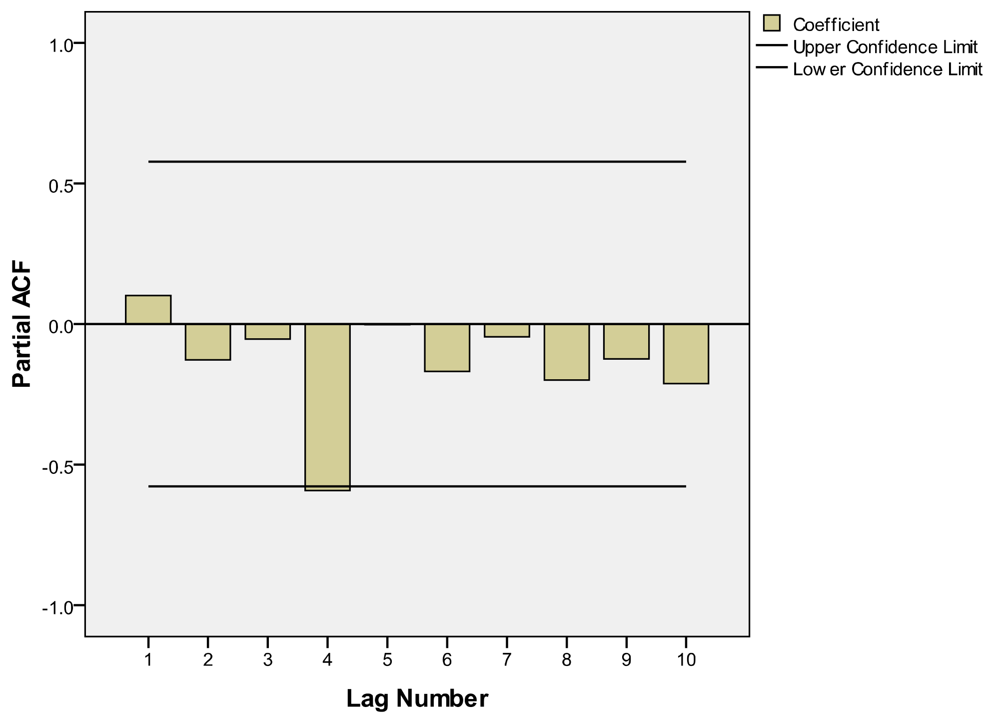

Fig. 4. The sample partial ACF of scaled residuals

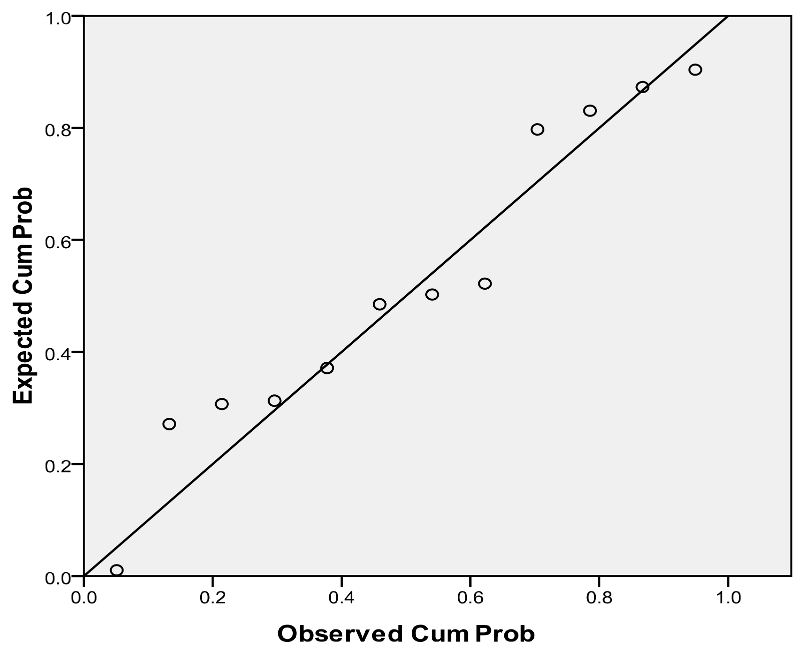

Fig. 5. The normal P-P plot of scaled residuals

6 Conclusion

In conclusion, it is reasonable to fit the yearly total number of accident data in Kuwait by using a lognormal diffusion model which is clearly supported by the residual analysis through the figures of the sample ACF and the sample partial ACF as well as the normal probability plot of the residuals $W(n)$. Therefore, this study provided a methodology capable and convenient to fit the traffic accident data. More specifically, the study departs from the traditional before-and-after regression techniques and the time series analysis and introduced a diffusion model that explicitly accounts for the variations in the total number of accidents.

In terms of future research, this methodology could be applied to determine the impact of installing traffic control devices on certain transportation facilities and the introduction of mandatory traffic laws.

Disclaimer

Some part of this manuscript was previously presented in the following conference.

Conference name: The Second International Conference on Computer Science,
Computer Engineering, and Education Technologies
Dates: September 8-10, 2015.
Location: Malaysia, Kuala Lumpur.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Rock SM. Compensation and the Illinois seat belt use law. Paper presented at 72nd Annual Transportation Research Board Meeting, Washington DC; 1993.

[2] Asch P, Levy D, Shea D, Bodenhorn D. Risk compensation and the effectiveness of safety belt use laws: A case study of New Jersey. Policy Sciences. 1991;24:181-197.

[3] Evans WN, Graham JD. Risk reduction or risk compensation? The case of mandatory safety belt use laws. Journal of Risk and Uncertainty. 1991;4:61-73.

[4] Garbacz C. Impact of the New Zealand seat belt laws. Economic Inquiry. 1991;29:310-316.

[5] Garbacz C. More evidence of the effectiveness of seat belt laws. Applied Economics. 1992;24:313-315.

[6] Transportation Research Board (TRB). Safety belts, airbag and child restraints. National Research Council, Washington, DC; 1989.

[7] Helfenstein U. When did a reduced speed limit shows an effect? Exploratory identification of intervention time. Accident Analysis and Prevention. 1990;22:79-87.

[8] Hamed MM, Al-Eideh BM, Al-Sharif MM. Traffic accidents under the effect of the Gulf crisis. Safety Science. 1999;33:59-68.

[9] Caliendo C, Guida M, Parisi A. A crash-prediction model for multilane roads. Accident Analysis and Prevention. 2007;39(4):657-670.

[10] Park ES, Lord D. Multivariate poisson-lognormal models for jointly modeling crash frequency by severity. Transportation Research Record. 2007;2019:1-6.

[11] Ma J, Kockelman K, Damien P. A multivariate Poisson-lognormal regression model for prediction of crash counts by severity, using Bayesian methods. Accident Analysis and Prevention. 2008;40(3): 964–975.

[12] Cafiso S, Graziano AD, Silvestro GD, La Cava G, Persaud B. Development of comprehensive accident models for two-lane rural highways using exposure, geometry, consistency and context variables. Accident Analysis and Prevention. 2010;42(4):1072-1079.

[13] Chen X, Li L, Zhang Y. A Markov model for headway/spacing distribution of road traffic. IEEE Transactions on Intelligent Transportation Systems. 2010;11(4):773–785.

[14] Lord D, Mannering F. The statistical analysis of crash-frequency data: A review and assessment of methodological alternatives. Transportation Research Part A. 2010;44(5):291-305.

[15] Malyshkina N, Mannering F. Zero-state Markov switching count-data models: An empirical assessment. Accident Analysis and Prevention4. 2010;2(1):122-130.

[16] Anastasopoulos P, Mannering F. An empirical assessment of fixed and random parameter logit models using crash- and non-crash-specific injury data. Accident Analysis and Prevention. 2011; 43(3):1140-1147.

[17] Jovanovic D, Backalic T, Basic S. The application of reliability models in traffic accident frequency analysis. Safety Science. 2011;49(8-9):1256-1251.

[18] Savolainen P, Mannering F, Lord D, Quddus M. The statistical analysis of crash-injury severities: A review and assessment of methodological alternatives. Accident Analysis and Prevention. 2011; 43(5):1666-1676.

[19] Wang C, Quddus MA, Ison SG. Predicting accident frequency at their severity levels and its application in site ranking using a two-stage mixed multivariate model. Accident Analysis and Prevention. 2011;43(6):1979-1990.

[20] Castro M, Paleti R, Bhat CR. A latent variable representation of count data models to accommodate spatial and temporal dependence: Application to predicting crash frequency at intersections. Transportation Research Part B. 2012;46(1):253-272.

[21] Geedipally S, Lord D, Dhavala S. The negative binomial-Lindley generalized linear model: Characteristics and application using crash data. Accident Analysis and Prevention. 2012;45:258-265.

[22] Aguero-Valverde J. Full Bayes poisson gamma, poisson lognormal and zero inflated random effects models: Comparing the precision of crash frequency estimates. Accident Analysis and Prevention. 2013;50:289-297.

[23] Bhat C, Born K, Sidharthan R, Bhat P. A count data model with endogenous covariates: Formulation and application to roadway crash frequency at intersections. Technical paper, Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin; 2013. Available:http://www.ce.utexas.edu/prof/bhat/ABSTRACTS/MNP_Count.pdf

[24] Caliendo C, De Guglielmo M, Guida M. A crash-prediction model for road tunnels. Accident Analysis and Prevention. 2013;55:107-115.

[25] Chiou YC, Fu C. Modeling crash frequency and severity using multinomial-generalized Poisson model with error components. Accident Analysis and Prevention. 2013;50:73-82.

[26] Deublein M, Schubert M, Adey B, Kohler J, Faber M. Prediction of road accidents: A Bayesian hierarchical approach. Accident Analysis and Prevention. 2013;51:274-291.

[27] Narayanamoorthy S, Paleti R, Bhat C. On accommodating spatial dependence in bicycle and pedestrian injury counts by severity level. Transportation Research Part B. 2013;55:245-264.

[28] Zou Y, Zhang Y, Lord D. Application of finite mixture of negative binomial regression models with varying weight parameters for vehicle crash data analysis. Accident Analysis and Prevention. 2013; 50:1042–1051.

[29] Brockwell PJ, Gani J, Resnick SI. Catastrophe processes with continuous state-space. Australian Journal of Statistics. 1983;25:208-226.

[30] Brockwell PJ. The extinction of birth, death and catastrophe process and of a related diffusion models. Advanced Applied Prob. 1985;17:42-52.

[31] Sorensen M. Likelihood methods for diffusions with jumps (Research report No.159). Dept. of Theoretical Statistics. Arhus University; 1988.

[32] Rrez RG, Romadn P, Torres F. Inference and first-passage times for the lognormal diffusion process with exogenous factors: Application to modelling in economics. Appl. Stochastic Models Bus. Ind. 1999;15:325-332.

[33] Tintner G, Patel RC. A log normal diffusion process applied to the economic development of India. Indian Economic Journal. 1965;13:465-474.

[34] Sprekel C. Warrant prices as indicators of expectations and preferences. Yale Economic Essays. 1961;1:178-231.

[35] Granger CWJ. Morgenstern. Spectral analysis of economic time series. Princeton University Press, Princeton, New Jersey; 1963.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/17707