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Analytical Approximate Solutions of Fractional Convection-Diffusion Equation by Means of Local Fractional Derivative Operators

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this article, the local fractional decomposition method (LFDM) is applied to obtain approximate the analytical solution of nonlinear fractional convection-diffusion. Numerical solutions obtained by local fractional decomposition method are compared with the exact solutions, revealing that the obtained solutions are of high accuracy. A new application of local fractional decomposition method (LFDM) was extended to reproduce the analytical solutions to this equation in the form of a series. It is shown that the solutions obtained by the LFDM are reliable, simple and that LFDM is an effective method for strongly nonlinear partial equations.

Keywords: Local fractional decomposition method; fractional convection-diffusion equation; Riemann-Liouville derivative.

1 Introduction

Local fractional calculus (LFC) was used in the modeling and processing of non-differentiable phenomena in various physical phenomena [1–15]. Some of these local fractional models can be listed as the local

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fractional Fokker- Planck equation (LFFPE) [1], the local fractional stress strain relations (LFSSR) [2], the local fractional heat conduction equation (LFHCE) [12], wave equations on the Cantor sets (WECSs)[14], local fractional Laplace equation (LFLE) [15] and Newtonian mechanics (NM) on fractals subset of real-line [16]. An approximate solution of the fractional diffusion equation which includes an absorbent term and external force was given by Das et al. [17]. Similar studies were made by the following: Momani and Yildirim [18] examined Fractional convection-diffusion equation with nonlinear source term, Yildirim and Kocak [19] studied space-time fractional advection-dispersion equation, Yildirim and Gulkanat [20] worked on fractional Zakharov-Kuznetsov equations, El-Shahed [21] studied an integro-differential equation, Siddiqui et al. [22] studied non-Newtonian flow, Yildirim [23] studied fractional PDEs in fluid mechanics and Kumar et al. [24] studied reaction-diffusion Brusselator system with fractional time derivative.

Successful applications of the homotopy perturbation method (HPM) [25-26], the Adomian decomposition method [27-29], the homotopy analysis method (HAM) [30] and the variational iteration method [31-33], which was given by Ji-Huan He, have been given on autonomous ordinary and partial differential equations and various other fields. The first application of variational iteration method on fractional differential equations was made by Ji-Huan He [34]. Jumarie [5] recently recommended a new modified Riemann-Liouville left derivative. Also, Adomian decomposition method, HPM and FVIM were used also by Momani [35] to solve nonlinear fractional convection-diffusion equation. Finally, a recent effective method called the Residual power series is used to solve important models, such as the time fractional Fisher [36] and other nonlinear fractional models arise in biology and physics [37-40].

In this study, the use of LFDM is extended to find analytical approximate solutions for the nonlinear fractional convection-diffusion problem

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^2 u(x,t)}{\partial x^2} - c\frac{\partial u(x,t)}{\partial x} + \Phi\big(u(x,t)\big) + f(x,t), \tag{1}$$

$$0 < x \le 1, 0 < \alpha \le 1, t > 0,$$

$$u(x,0) = h(x), 0 < x \le 1, \tag{2}$$

here $\Phi(u)$ is an appropriate nonlinear function of u selected as potential energy, α is the parameter that describes time-fractional derivative order and c is a constant. We consider the fractional derivative in the modified Riemann-Liouville derivative. $u(x,t)$ is a causal function of time which means it vanishes for $t < 0$. The fields of science and engineering have wide usage of the convection-diffusion equations for the mathematical modeling of computational simulations in oil reservoir simulations, mass and energy transport and global weather productions, where diffusion and convection propagates the initially discontinuous profile, the latter with a speed of c .

This study aims to extend the use of LFDM in solving fractional nonlinear convection-diffusion equations with modified Riemann-Liouville derivative.

The paper organization is as follows:

> Definitions related to local fractional calculus theory are given in Section 2. Solution procedure of local fractional decomposition method is defined to emphasize the inefficiency of this method in Section 3. The method is used on the problem (19)-(30) and graphs are given for numerical simulations, respectively in Section 4. Section 5 includes the conclusions.

2 Basic Definitions

Let us recall some definitions and properties of local fractional continuity (LFC), local fractional derivative (LFD) and local fractional integral (LFI) of non-differential functions [1-10, 41-44].

Definition 1 Assume the relation below exists [9,41-44]

$$|f(u) - f(u_0)| < \varepsilon^\alpha \tag{3}$$

with $|u - u_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in \mathbb{R}$. Then $f(u)$ is local fractional continuous at $u = u_0$ which is denoted by $\lim_{u\to u_0} f(u) = f(u_0)$. If $f(u)$ is local fractional continuous on the interval (a, b), it is denoted by

$$f(u) \in C_\alpha(a, b). \tag{4}$$

Definition 2 $f(x)$ is a non-differentiable function of exponent $0 < \alpha \le 1$ if it is a Hölder function of exponent α, i.e.

$$|f(u) - f(v)| < C|u - v|^\alpha \tag{5}$$

for $u, v \in X$ [9, 41-44].

Definition 3 $f(x)$ is said to be continuous of order α (or α continuous) with $0 < \alpha \le 1$ if the following relation is satisfied [9,41-44], $|f(u) - f(u_0)| < \varepsilon^\alpha$,

$$f(u) - f(u_0) = o((u - u)^\alpha). \tag{6}$$

(3) is the standard definition for local fractional continuity (LFC) when compared with (6) and (5) is the unified local fractional continuity (ULFC).

Definition 4. For $f(u) \in C_\alpha(a, b)$, LFD of $f(x)$ of order α at $u = u_0$, is given by [9,41-44]:

$$f^{(\alpha)}(u_0) = \left.\frac{d^\alpha f(u)}{du^\alpha}\right|_{u=u_0} = \lim_{u\to u_0} \frac{\Delta^\alpha(f(u) - f(u_0))}{(u - u_0)^\alpha}, 0 < \alpha \le 1, \tag{7}$$

where $\Delta^\alpha(f(u) - f(u_0)) \cong \Gamma(1 + \alpha)\Delta(f(u) - f(u_0))$. For any $u \in (a, b)$, there

$f^{(\alpha)}(u) = D_u^\alpha f(u)$ exists, denoted by $f(u) \in D_u^\alpha(a, b)$. LFD of higher order can be expressed as follows:

$$f^{(k\alpha)}(u) = \underbrace{D_u^\alpha \ldots D_u^\alpha f(u)}_{k\ times},$$

and the higher ordered local fractional partial derivative (LFPD) is shown as:

$$\frac{\partial^{(k\alpha)} f(u)}{\partial u^{k\alpha}} = \underbrace{\frac{\partial^\alpha}{\partial u^\alpha} \ldots \frac{\partial^\alpha}{\partial u^\alpha} f(u).}_{k\ times}$$

Definition 5. For $f(u) \in C_\alpha(a, b)$, local fractional integral (LFI) of $f(u)$ of order α within interval $[a, b]$ is given by [41-44]:

$${}_aI_b^\alpha f(u) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \tag{8}$$

$$= \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t\to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, \ \ 0 < \alpha \le 1,$$

where $\Delta t_j = t_{j+1} - t_j, \Delta t = max\{\Delta t_1, \Delta t_2, \Delta t_3, ...\}$ and $[\Delta t_j, \Delta t_{j+1}]$, $j = 0,1, ..., N-1, t_0 = a, t_N = b$, is a partition of the interval $[a, b]$. For any $u \in (a, b)$, there exists ${}_aI_b^{\alpha} f(u)$, denoted by $f(u) \in I_u^{(\alpha)}(a, b)$. If $f(u) = D_u^{\alpha} f(a, b)$, or $I_u^{(\alpha)}(a, b)$, we have $f(u) \in C_{\alpha}(a, b)$.

For any $f(u) \in C_{\alpha}(a, b)$, $0 < \alpha \leq 1$, local fractional multiple integrals (LFMIs) have the form:

$$ {}_{u_0}I_u^{(k\alpha)} f(u) = \overbrace{{}_{u_0}I_u^{(\alpha)} \cdots {}_{u_0}I_u^{(\alpha)} f(u)}^{k\ times}, $$

for $0 < \alpha \leq 1$, $f^{(k\alpha)}(u) \in C_{\alpha}^k(a, b)$, then we have

$$ \left({}_{u_0}I_u^{(k\alpha)} f(u)\right)^{(k\alpha)} = f(u), $$

where ${}_{u_0}I_u^{(k\alpha)} f(u) = \underbrace{{}_{u_0}I_u^{(\alpha)} \cdots {}_{u_0}I_u^{(\alpha)} f(u)}_{k\ times}$ and $f^{(k\alpha)}(u) = \underbrace{D_u^{\alpha} \dots D_u^{\alpha} f(u)}_{k\ times}$.

Definition 6. In the fractional space, the Mittag-Leffler function (MLF) is given by (see [9,41-44])

$$ E_{\alpha}(u^{\alpha}) = \sum_{n=0}^{\infty} \frac{u^{(n\alpha)}}{\Gamma(1+n\alpha)}, 0 < \alpha \leq 1. \tag{9} $$

Some useful formulas of LFD were summarized [9,44] as follows:

$$ \frac{d^{\alpha} u^{n\alpha}}{du^{\alpha}} = \frac{\Gamma(1+n\alpha) u^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)}, \tag{10} $$

$$ \frac{d^{\alpha} E_{\alpha}(u^{\alpha})}{du^{\alpha}} = E_{\alpha}(u^{\alpha}), \tag{11} $$

$$ \frac{d^{\alpha} E_{\alpha}(nu^{\alpha})}{du^{\alpha}} = nE_{\alpha}(nu^{\alpha}), \tag{12} $$

$$ \frac{1}{\Gamma(1+\alpha)} \int_a^b E_{\alpha}(u^{\alpha})(du)^{\alpha} = E_{\alpha}(b^{\alpha}) - E_{\alpha}(a^{\alpha}), \tag{13} $$

$$ \boldsymbol{\frac{1}{\Gamma(1+\alpha)} \int_a^b u^{n\alpha} (du)^{\alpha} = \frac{\Gamma(1+n\alpha)\left(b^{(n+1)\alpha} - a^{(n+1)\alpha}\right)}{\Gamma(1+(n+1)\alpha)}}. \tag{14} $$

3 Local Fractional Decomposition Method

We demonstrate the local fractional decomposition method solution procedure by examining the following fractional differential equation [18,43-44]:

$$ \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^2 u(x,t)}{\partial x^2} - c\frac{\partial u(x,t)}{\partial x} + \Phi\big(u(x,t)\big) + f(x,t), \tag{15} $$

$$ 0 < x \leq 1, 0 < \alpha \leq 1, t > 0, $$

A local fractional differential operator form for Eq.(15) can be built by the LFDM as follows

$$ L_t^{(\alpha)} u(x,t) = u_{xx}(x,t) - u_x(x,t) + \Phi(u(x,t)) + f(x,t), \tag{16} $$

where $0 < \alpha \leq 1$, $u(x,t)$ is a local fractional continuous function. An application of the inverse operator $L_t^{(-\alpha)}$ on both sides of (16) gives

$$\begin{cases} u_{n+1}(x,t) = L_t^{(-\alpha)}\left[\frac{\partial^2}{\partial x^2}u_n(x,t) - \frac{\partial}{\partial x}u_n(x,t) + \Phi(u_n(x,t)) + f(x,t)\right], n \geq 0 \\ u_0(x,t) = u(x,0). \end{cases} \tag{17}$$

We can obtain successive approximations $u_{n+1}(x,t)$, $n \geq 0$ of the solution $u(x,t)$ by the use of a selective function u_0. Generally, the initial values are considered for the zeroth approximation u_0. Thus, the exact solution can be obtained from

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{18}$$

Hence, the condition

$$|f(x) - f(x_0)| < \varepsilon^{\alpha},$$

can be found, where the fractional dimension of $f(x)$ is α for any $x \in (a,b)$.

4 Applications

This section presents the solutions of nonlinear fraction differential equations by an application of the local fractional decomposition method (LFDM).

Examples 4.1. Let's examine the following nonlinear fractional convection-diffusion equation where $0 < \alpha \leq 1, c = 1, 0 < x \leq 1, t > 0$,

and $\Phi(u) = u\frac{\partial^2 U}{\partial x^2} - u^2 + u$. We get

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial x} + u(x,t)\frac{\partial^2 u(x,t)}{\partial x \partial t} - u(x,t)^2 + u(x,t) \tag{19}$$

with the initial condition

$$u(x,0) = e^x, \text{ [18,35,45]}. \tag{20}$$

Using Eq. (20), the recurrence relation is found as:

$u_0(x,t) = u(x,0)$ and

$$u_{n+1}(x,t) = L_t^{(-\alpha)}\begin{bmatrix} \frac{\partial^2}{\partial x^2}u_n(x,t) - \frac{\partial}{\partial x}u_n(x,t) \\ +u_n(x,t)\frac{\partial^2}{\partial x^2}u_n(x,t) - u_n(x,t)^2 + u_n(x,t) \end{bmatrix} n \geq 0. \tag{21}$$

With the recursive relation (21) and the condition in (20), the following results are obtained:

$$u_0(x,t) = e^x, \tag{22}$$

$$u_1(x,t) = L_t^{(-\alpha)}\begin{bmatrix} \frac{\partial^2}{\partial x^2}u_0(x,t) - \frac{\partial}{\partial x}u_0(x,t) \\ +u_0(x,t)\frac{\partial^2}{\partial x^2}u_0(x,t) - u_0(x,t)^2 + u_0(x,t) \end{bmatrix} = \frac{e^x t^\alpha}{\Gamma(1+\alpha)}, \tag{23}$$

$$u_2(x,t) = L_t^{(-\alpha)}\begin{bmatrix} \frac{\partial^2}{\partial x^2}u_1(x,t) - \frac{\partial}{\partial x}u_1(x,t) \\ +u_1(x,t)\frac{\partial^2}{\partial x^2}u_1(x,t) - u_1(x,t)^2 + u_1(x,t) \end{bmatrix} = \frac{e^x t^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{24}$$

$$u_3(x,t) = L_t^{(-\alpha)}\begin{bmatrix} \frac{\partial^2}{\partial x^2}u_2(x,t) - \frac{\partial}{\partial x}u_2(x,t) \\ +u_2(x,t)\frac{\partial^2}{\partial x^2}u_2(x,t) - u_2(x,t)^2 + u_2(x,t) \end{bmatrix} = \frac{e^x t^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{25}$$

$$\vdots,$$

$$u_n(x,t) = L_t^{(-\alpha)}\begin{bmatrix} \frac{\partial^2}{\partial x^2}u_{n-1}(x,t) - \frac{\partial}{\partial x}u_{n-1}(x,t) \\ +u_{n-1}(x,t)\frac{\partial^2}{\partial x^2}u_{n-1}(x,t) - u_{n-1}(x,t)^2 + u_{n-1}(x,t) \end{bmatrix} = \frac{e^x t^{n\alpha}}{\Gamma(1+n\alpha)} \tag{26}$$

Hence, the approximate solution can be given in a series form as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = e^x\left(1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right) = e^x \sum_{n=0}^{\infty} \frac{t^{(n\alpha)}}{\Gamma(1+n\alpha)} \tag{27}$$

And the corresponding exact solution would be

$$u(x,t) = e^x E_\alpha(t^\alpha). \tag{28}$$

For $\alpha = 1$ we would have

$$u(x,t) = e^x \sum_{k=0}^{\infty} \frac{\left(t^k\right)}{\Gamma(k+1)} = e^{x+t} \tag{29}$$

as the exact solution to the nonlinear convection-diffusion equation. Fig. 1 shows the approximate solutions for the nonlinear convection-diffusion equation (19) obtained by using local fractional decomposition method (LFDM).

The approximate solution of Eq. (19) is plotted in Fig. 2 for $\alpha = 0.9, 0.8, 0.7, 0.6$. The effect of a on $u(x,t)$ is demonstrated in the Figs. 3 and 4.

By considering $x = 0.5$ in the numerical results of Figs. 3 and 4, it can be said that a decrease in the fractional order α results in an increase in the function. Five sequential values $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ are demonstrated in Figs. 3 and 4.

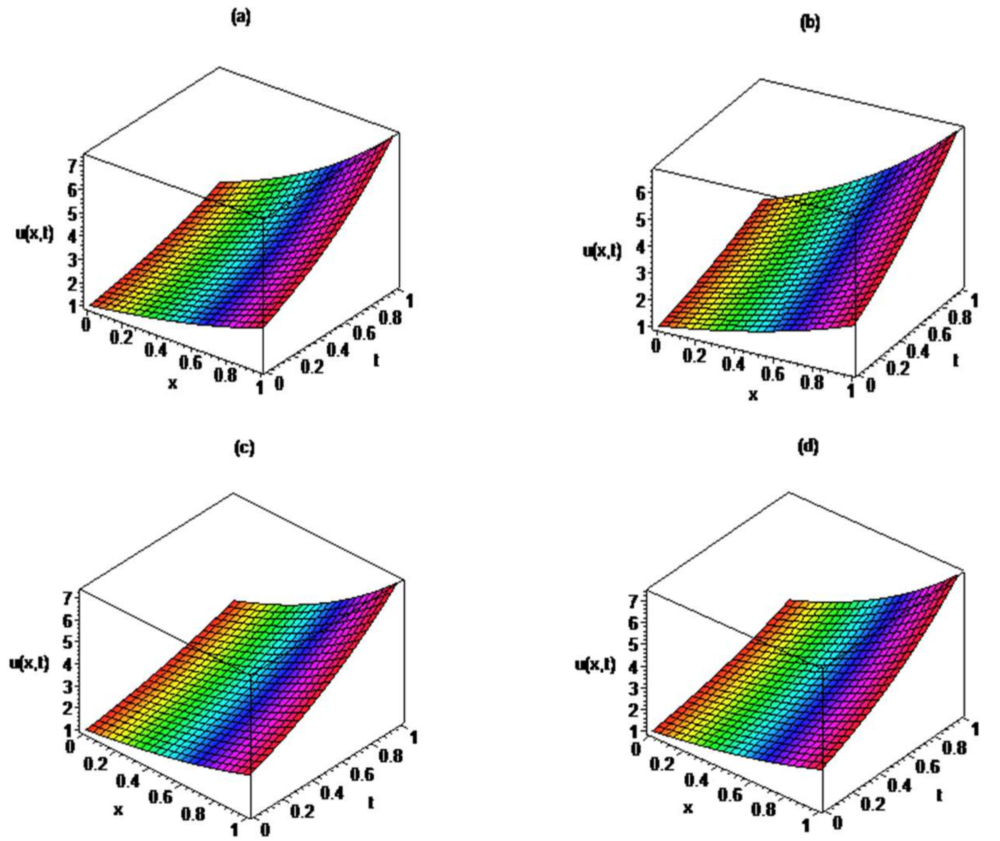

Fig. 1. The surface indicates the solution $u(x,t)$ of (19) for $\alpha = 1$. (a) Exact solution (b) 2-iterate LFD approximate solution, (c) 3-iterate LFD approximate solution and (d) 4-iterate LFD approximate solution

Table 1. Numerical values when $\alpha = 0.5, 0.75, 1$ for $u(x,t)$ obtained using the LFDM

t	x	u_{4LFDM}			u_{Exact}	u_{Error}
		α	α	α	α	
0.2	0	1.791910038	1.404517379	1.221400000	1.221402758	0.2758e-5
	0.25	2.300858034	1.803436013	1.568308644	1.568312185	0.3541 e-5
	0.50	2.954360195	2.315657678	2.013748160	2.013752707	0.4547e-5
	0.75	3.793473581	2.973363315	2.585703821	2.585709659	0.5838 e-5
	1	4.870916494	3.817874068	3.320109425	3.320116923	0.7498 e-5
0.4	0	2.383956218	1.798154972	1.491733334	1.491824698	0.000091364
	0.25	3.061060377	2.308876688	1.915423516	1.915540829	0.000117313
	0.50	3.930479326	2.964656351	2.459452478	2.459603111	0.000150633
	0.75	5.046835354	3.806694106	3.157999493	3.158192910	0.000193417
	1	6.480264866	4.887891984	4.054951614	4.055199967	0.000248353
0.6	0	3.003654242	2.251671269	1.821400000	1.822118800	0.000718800
	0.25	3.856768391	2.891203140	2.338723895	2.339646852	0.000922957
	0.50	4.952188640	3.712378316	3.002980923	3.004166024	0.001185101
	0.75	6.358736081	4.766788115	3.855903831	3.857425531	0.001521700
	1	8.164778744	6.120677093	4.951078522	4.953032424	0.001953902

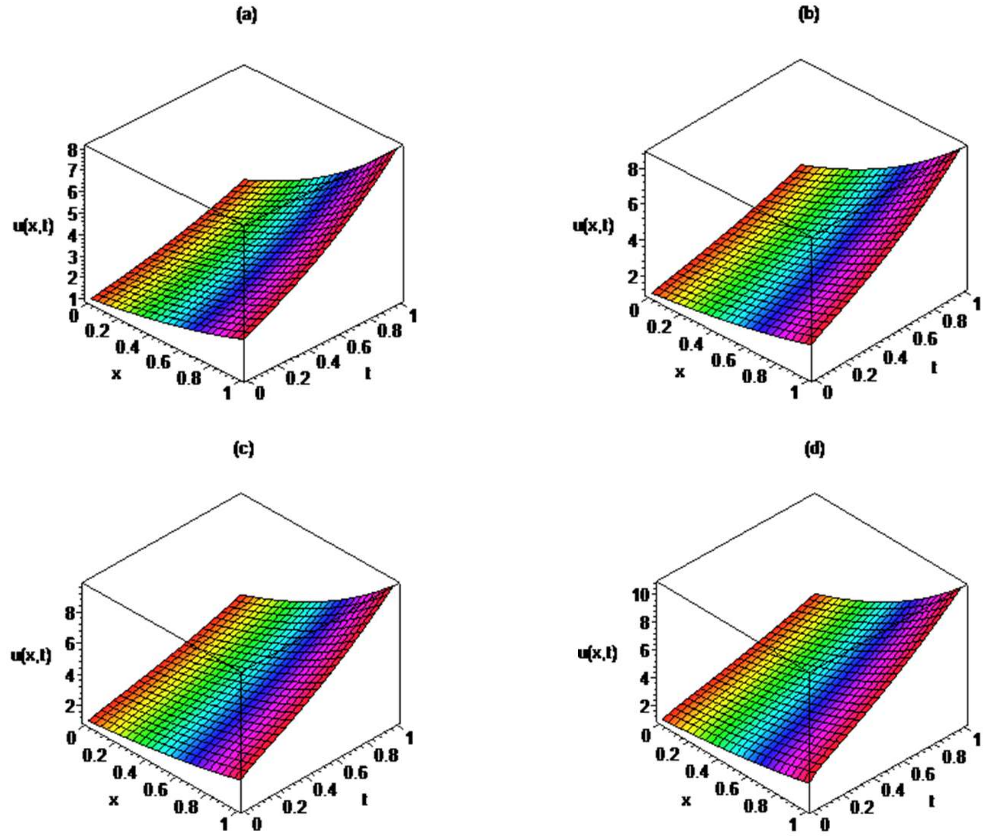

Fig. 2. The surface indicates the solution $u(x,t)$ of (19) (a) 4-iterate LFD approximate solution for $\alpha = 0.9$ (b) 4-iterate LFD approximate solution for $\alpha = 0.8$ (c) 4-iterate LFD approximate solution for $\alpha = 0.7$ and (d) 4-iterate LFD approximate solution for $\alpha = 0.6$

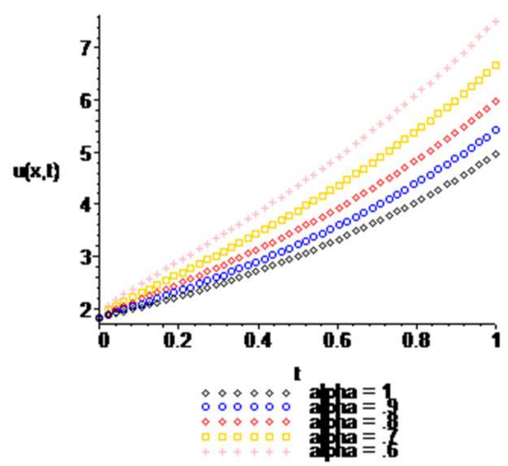

Fig. 3. 5-iterate LFD approximate solution for $x = 0.6$

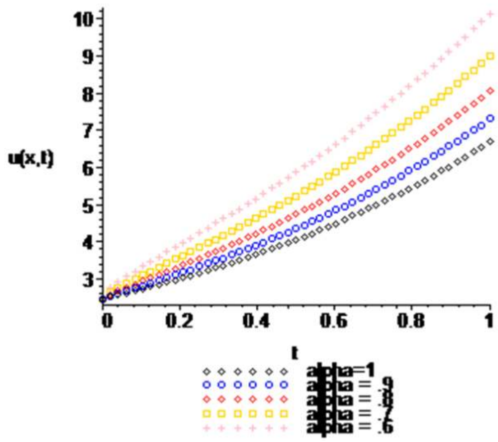

Fig. 4. 5-iterate LFD approximate solution for $x = 0.9$

Solution of (19) is found by using Adomian decomposition method in [35], FVIM in [45] and HPM in [18]. It is seen that the present algorithm for Local fractional decomposition method (LFDM) works with considerable efficiency, simplicity and reliability. The results obtained from LFDM are in full accordance with modified variational iteration method(FVIM), ADM and HPM.

Examples 4.2. Consider the non-homogenous nonlinear fractional convection-diffusion equation for $0 < \alpha \leq 1, c = 1, 0 < x \leq 1, t > 0$,

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial x} + \frac{\partial u(x,t)}{\partial t}\frac{\partial u(x,t)}{\partial x} + u(x,t)\frac{\partial^2 u(x,t)}{\partial x \partial t} - 2x \tag{30}$$

with the initial condition [18,35,45]

$$u(x,0) = x^2. \tag{31}$$

By LFAMD, the recurrence relation reads as follows:

$u_0(x,t) = u(x,0)$ and

$$u_{n+1}(x,t) = L_t^{(-\alpha)}\begin{bmatrix}\frac{\partial^2}{\partial x^2}u_n(x,t) - \frac{\partial}{\partial x}u_n(x,t) + \frac{\partial}{\partial t}u_n(x,t)\frac{\partial}{\partial x}u_n(x,t) \\ +u_n(x,t)\frac{\partial^2}{\partial x^2}u_n(x,t) - 2x\end{bmatrix} n \geq 0. \tag{32}$$

Using (32) and the condition (31), the followings results are obtained:

$$u_0(x,t) = x^2, \tag{33}$$

$$u_1(x,t) = L_t^{(-\alpha)}\begin{bmatrix}\frac{\partial^2}{\partial x^2}u_0(x,t) - \frac{\partial}{\partial x}u_0(x,t) + \frac{\partial}{\partial t}u_0(x,t)\frac{\partial}{\partial x}u_0(x,t) \\ +u_0(x,t)\frac{\partial^2}{\partial x^2}u_0(x,t) - 2x\end{bmatrix}, \tag{34}$$

$$= \frac{(2-4x)t^\alpha}{\Gamma(1+\alpha)},$$

$$u_2(x,t) = L_t^{(-\alpha)}\begin{bmatrix}\frac{\partial^2}{\partial x^2}u_1(x,t) - \frac{\partial}{\partial x}u_1(x,t) + \frac{\partial}{\partial t}u_1(x,t)\frac{\partial}{\partial x}u_1(x,t) \\ +u_1(x,t)\frac{\partial^2}{\partial x^2}u_1(x,t) - 2x\end{bmatrix} \tag{35}$$

$$= \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{(32x-16)\alpha\Gamma(2\alpha)t^{3\alpha-1}}{\Gamma(1+\alpha)^2\Gamma(3\alpha)} - \frac{2xt^{\alpha}}{\Gamma(1+\alpha)},$$

$$u_3(x,t) = L_t^{(-\alpha)}\left[\begin{array}{c}\frac{\partial^2}{\partial x^2}u_2(x,t) - \frac{\partial}{\partial x}u_2(x,t) + \frac{\partial}{\partial t}u_2(x,t)\frac{\partial}{\partial x}u_2(x,t) \\ +u_2(x,t)\frac{\partial^2}{\partial x^2}u_2(x,t) - 2x\end{array}\right] \tag{36}$$

$$= -\frac{32\alpha\Gamma(2\alpha)t^{4\alpha-1}}{\Gamma(\alpha+1)\Gamma(4\alpha)} + \frac{256\alpha^2\Gamma(2\alpha)\Gamma(5\alpha-1)t^{6\alpha-2}}{\Gamma(\alpha+1)^2\Gamma(3\alpha)\Gamma(1+2\alpha)\Gamma(6\alpha-1)} - \frac{16\alpha\Gamma(3\alpha)t^{4\alpha-1}}{\Gamma(1+2\alpha)\Gamma(1+\alpha)\Gamma(4\alpha)} + \frac{32(32x-16)\alpha^2(3\alpha-1)\Gamma(2\alpha)^2\Gamma(6\alpha-2)t^{7\alpha-3}}{\Gamma(\alpha+1)^4\Gamma(3\alpha)^2\Gamma(7\alpha-2)} - \frac{2(32x-16)\alpha(3\alpha-1)\Gamma(2\alpha)\Gamma(4\alpha-1)t^{5\alpha-2}}{\Gamma(\alpha+1)^3\Gamma(3\alpha)\Gamma(5\alpha-1)} - \frac{64x\alpha^2\Gamma(2\alpha)\Gamma(4\alpha-1)t^{5\alpha-2}}{\Gamma(\alpha+1)^3\Gamma(3\alpha)\Gamma(5\alpha-1)} + \frac{4x\alpha\Gamma(2\alpha)t^{3\alpha-1}}{\Gamma(\alpha+1)\Gamma(3\alpha)} - \frac{2xt^{\alpha}}{\Gamma(\alpha+1)},$$

$\vdots$,

Hence, the approximate solution can be given in a series form as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = x^2 + \frac{(2-8x)t^{\alpha}}{\Gamma(1+\alpha)} + \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{(32x-16)\alpha\Gamma(2\alpha)t^{3\alpha-1}}{\Gamma(1+\alpha)^2\Gamma(3\alpha)} - = -\frac{32\alpha\Gamma(2\alpha)t^{4\alpha-1}}{\Gamma(\alpha+1)\Gamma(4\alpha)} + \frac{256\alpha^2\Gamma(2\alpha)\Gamma(5\alpha-1)t^{6\alpha-2}}{\Gamma(\alpha+1)^2\Gamma(3\alpha)\Gamma(1+2\alpha)\Gamma(6\alpha-1)} - \frac{16\alpha\Gamma(3\alpha)t^{4\alpha-1}}{\Gamma(1+2\alpha)\Gamma(1+\alpha)\Gamma(4\alpha)} + \frac{32(32x-16)\alpha^2(3\alpha-1)\Gamma(2\alpha)^2\Gamma(6\alpha-2)t^{7\alpha-3}}{\Gamma(\alpha+1)^4\Gamma(3\alpha)^2\Gamma(7\alpha-2)} - \frac{2(32x-16)\alpha(3\alpha-1)\Gamma(2\alpha)\Gamma(4\alpha-1)t^{5\alpha-2}}{\Gamma(\alpha+1)^3\Gamma(3\alpha)\Gamma(5\alpha-1)} - \frac{64x\alpha^2\Gamma(2\alpha)\Gamma(4\alpha-1)t^{5\alpha-2}}{\Gamma(\alpha+1)^3\Gamma(3\alpha)\Gamma(5\alpha-1)} + \frac{4x\alpha\Gamma(2\alpha)t^{3\alpha-1}}{\Gamma(\alpha+1)\Gamma(3\alpha)} \ldots = \tag{37}$$

For $\alpha = 1$ we would have [44]

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = x^2 + 2t - 4tx + 2t^2 + 4tx - 12x^2t + 16xt^2 - 8t^2 + \cdots, \tag{38}$$

The exact solution of (38) can be found by omitting the noise terms and keeping the non-noise terms as

$$u(x,t) = x^2 + 2t \tag{39}$$

and this can be easily confirmed. Formal proof can be found in [46].

Table 2. Approximate solution of (30) at t=1 for various values of the order α

	[35]				u_{3LFDM}			
x	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.9$	$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.9$	$\alpha = 1$
0.0	5.00328	3.48585	2.97494	2.71828	-5.23681	-5.92718	-4.92124	-4
0.1	5.52948	3.85246	3.28782	3.00417	-3.60512	-4.34755	-3.65775	-2.99000
0.2	6.11102	4.25762	3.63360	3.32012	-1.95344	-2.74793	-2.37427	-1.96000
0.3	6.75372	4.70540	4.01575	3.66930	-0.28175	-1.12830	-1.07078	-0.91000
0.4	7.46401	5.20027	4.43809	4.05520	1.40993	0.511319	0.25269	0.160000
0.5	8.24901	5.74710	4.49048	4.48169	3.12162	2.17094	1.59618	1.25000
0.6	9.11656	6.35163	5.42069	4.95303	4.85330	3.85057	2.95966	2.36000
0.7	10.07540	7.01964	5.99079	5.47395	6.60499	5.55019	4.34315	3.49000
0.8	11.13500	7.75790	6.62085	6.04965	8.37668	7.26982	5.74663	4.64000
0.9	12.30812	8.57380	7.31717	6.68509	10.1683	9.00945	7.17012	5.81000

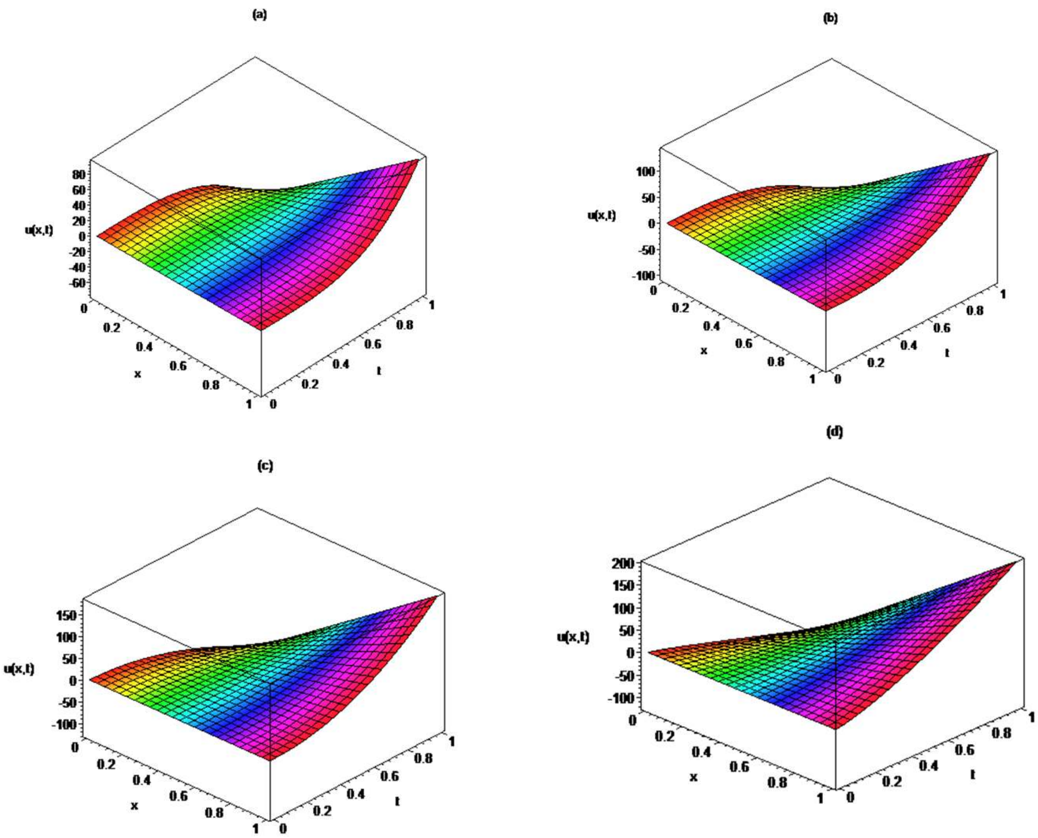

Fig. 5. The surface shows the solution $u(x,t)$ of (30) (a) 3-iterate LFD approximate solution for $\alpha = 0.9$ (b) 3-iterate LFD approximate solution for $\alpha = 0.8$ (c) 3-iterate LFD approximate solution for $\alpha = 0.7$ and (d) 3-iterate LFD approximate solution for $\alpha = 0.6$

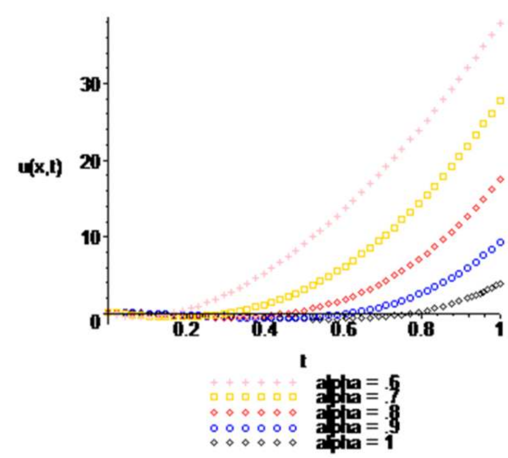

Fig. 6. 3-iterate LFD approximate for $x = 0.5$

Lastly, Fig. 5 shows the solution surfaces of the non-homogenous nonlinear fractional convection-diffusion equation for various α values. From the graphical results in Figs. 5 and 6, it can be seen the effect of α on the function $u(x,t)$. Certainly, $u(x,t)$ increases with an increase in t for $\alpha = 1,0.9,0.8,0.7,0.6$ ..

Solution of 30 is found by using Adomian decomposition method in [35], FVIM in [45] and HPM in [18]. The results in Fig. 5 are in accordance with the results of the Adomian decomposition method and HPM.

5 Conclusions

Local fractional decomposition method (LFDM) has been successfully employed for finding the solutions of nonlinear problems, ordinary, partial, fractional and integral equations. This study used the Local fractional decomposition method having integral with respect to $(d\tau)^{\alpha}$ which had been used by Jumarie for the first time. Results show that the method is a powerful tool and is meaningful for the solutions of nonlinear fractional differential equations. The results of the two different examples obtained by local fractional decomposition method having integral with respect to $(d\tau)^{\alpha}$ are in perfect accordance with the results from classical VIM, MVIM, GDTM, HPM and Adomian decomposition method which can be found in the referred studies.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Kolwankar KM, Gangal AD. Local fractional Fokker-Planck equation. Physical Review Letters. 1998;80:214.

[2] Carpinteri A, Chiaia B, Cornetti P. Static–kinematic duality and the principle of virtual work in the mechanics of fractal media. Computer Methods in Applied Mechanics and Engineering. 2001;191:3-19.

[3] Adda FB, Cresson J. About non-differentiable functions. Journal of Mathematical Analysis and Applications. 2001;263:721–737.

[4] Babakhani A, Daftardar VG. On calculus of local fractional derivatives. Journal of Mathematical Analysis and Applications. 2002;270:66-79.

[5] Jumarie G. Table of some basic fractional calculus formulae derived from a modified Riemann–Liouvillie derivative for non-differentiable functions. Applied Mathematics Letters. 2009;22:378-385.

[6] Chen W, Sun H, Zhang X, Korosak D. Anomalous diffusion modeling by fractal and fractional derivatives. Computers and Mathematics with Applications. 2010;59:1754-1758.

[7] Merdan M. Solutions of time-fractional reaction-diffusion equation with modified Riemann-Liouville derivative. International Journal of Physical Sciences. 2012;7(15):2317-2326.

[8] Merdan M, Gökdoğan A, Yıldırım, Mohyud-Din ST. Numerical simulation of fractional Fornberg-Whitham equation by differential transformation method. Abstract and Applied Analysis. 2012;Article ID 965367.

[9] Yang XJ. Local fractional functional analysis and its applications. Asian Academic Publisher, Hong Kong, PRC; 2011.

[10] Yang XJ. Advanced local fractional calculus and its applications. World Science Publisher, New York, USA; 2012.

[11] Kilbas AA, Srivastava HM, Trujillo JJ. Theory and applications of fractional differential equations. Amsterdam, Elsevier; 2006.

[12] Yang XJ, Baleanu D. Fractal heat conduction problem solved by local fractional variation iteration method. Thermal Science. 2013;17:625-628.

[13] Yang XJ, Srivastava HM, He JH, Baleanu D. Cantor-type cylindrical-coordinate method for differential equations with local fractional derivatives. Physics Letters A. 2013;377:1696–1700.

[14] Ahmad J, Mohyud-Din ST. Solving wave and diffusion equations on Cantor sets Proc. Pakistan Acad. Sci. 2015;52:71-77.

[15] Yang YJ, Baleanu D, Yang XJ. A local fractional variational iteration method for Laplace equation within local fractional operators. Abstract and Applied Analysis. 2013;Art. ID 202650.

[16] Golmankhaneh AK, Fazlollahi V, Baleanu D. Newtonian mechanics on fractals subset of real-line. Romanian Reports in Physics. 2013;65:84-93.

[17] Das S, Gupta PK. An approximate analytical solution of the fractional diffusion equation with absorbent term and external force by homotopy perturbation method. Z. Naturforsch. 2010;65:182-190.

[18] Momani S, Yildirim A. Analytical approximate solutions of the fractional convection-diffusion equation with nonlinear source term by he's homotopy perturbation method. Int. J. Comput. Math. 2010;87:1057-1065.

[19] Yildirim A, Kocak H. Homotopy perturbation method for solving the space–time fractional advection-dispersion equation. Adv. Water Resour. 2009;*32:*1711-1716.

[20] Yildirim A, Gulkanat Y. Analytical approach to fractional Zakharov-Kuznetsov equations by he's homotopy perturbation method. Commun. Theor. Phys. 2010;53:1005.

[21] El-Shahed M. Application of He's homotopy perturbation method to Volterra's integro-differential equation. Int. J. Nonlinear Sci. Numer. Simul. 2005;6:163-168.

[22] Siddiqui AM, Mahmood R, Ghori QK. Thin film flow of a third grade fluid on a moving belt by he's homotopy perturbation method. Int. J. Nonlinear Sci. Numer. Simul. 2006;7:7-14.

[23] Yildirim A. Analytical approach to fractional partial differential equations in fluid mechanics by means of the homotopy perturbation method. Internat. J. Numer. Methods Heat Fluid Flow. 2010;20: 186-200.

[24] Kumar S, Khan Y, Yildirim A. A mathematical modeling arising in the chemical systems and its approximate numerical solution. Asia-Pac. J. Chem. Eng; 2011. (In press).

[25] Ganji ZZ, Ganji DD, Jafari H, Rostamian M. Application of the homotopy perturbation method to coupled system of partial differential equations with time fractional derivatives. Topol. Methods Nonlinear Anal. 2008;31:341-348.

[26] Yildirim A. An algorithm for solving the fractional nonlinear Schrödinger equation by means of the homotopy perturbation method. Int. J. Nonlinear Sci. Numer. Simul. 2009;10:445-451.

[27] Adomian G. A review of the decomposition method in applied mathematics. J. Math. Anal. Appl. 1998;135:501-544.

[28] Adomian G. Solving frontier problems of physics: The decomposition method. Kluwer Academic Publishers, Boston; 1994.

[29] Wazwaz AM. A new algorithm for calculating Adomian polynomials for nonlinear operators. Appl. Math. Comput. 2000;111:53-69.

[30] Dehghan M, Manafian J, Saadatmandi A. Solving nonlinear fractional partial differential equations using the homotopy analysis method. Numer. Met. for Partial Diff. Equ. 2010;26:448-479.

[31] He JH. Variational iteration method for delay differential equations. Commun. Nonlinear Sci. Numer. Simul. 1997;2:235-236.

[32] He JH. Variational iteration method- a kind of non-linear analytical technique: Some examples. Int. J. Nonlinear Mech. 1999;34:699-708.

[33] He JH. Variational iteration method for autonomous ordinary differential systems. Appl. Math. Comput. 2000;114:115-123.

[34] He JH. Some applications of nonlinear fractional differential equations and their approximations. Bull. Sci. Technol. 1999;15:86-90.

[35] Momani S. An algorithm for solving the fractional convection–diffusion equation with nonlinear source term. Commun. Nonlinear. Sci. Numer. Simul. 2007;12:1283-1290.

[36] Alquran M, Al-Khaled K, Sardar T, Chattopadhyay J. Revisited Fisher's equation in a new outlook: A fractional derivative approach. Physica A: Statistical Mechanics and its Applications. 2015;438:81-93.

[37] Alquran M, Al-Khaled K, Chattopadhyay J. Analytical solutions of fractional population diffusion model: Residual power series. Nonlinear Studies. 2015;22(1):31-39.

[38] Alquran M. Analytical solutions of fractional foam drainage equation by residual power series method. Mathematical Sciences. 2014;8(4):153-160.

[39] Alquran M. Analytical solution of time-fractional two-component evolutionary system of order 2 by residual power series method. Journal of Applied Analysis and Computation. 2015;5(4):589-599.

[40] Jaradat HM, Al-Shara S, Khan QJA, Alquran M, Al-Khaled K. Analytical solution of time-fractional Drinfeld-Sokolov-Wilson system using residual power series method. IAENG International Journal of Applied Mathematics. 2016;46(1):64-70.

[41] Ahmad J, Mohyud-Din ST, Srivastava HM, Yang XJ. Analytic solutions of the Helmholtz and Laplace equations by using local fractional derivative operators. Waves Wavelets Fractals Adv. Anal. 2015;1:22–26.

[42] Yang XJ, Zhang YD. A new adomian decomposition procedure scheme for solving local fractional Volterra integral equation, Advances in Information Technology and Management. 2012;1(4):158-161.

[43] Yang XJ. Local fractional integral transforms. Progress in Nonlinear Science. 2011;4(4):1.

[44] Faraz N, Khan Y, Jafari H, Yildirim A, Madani M. Fractional variational iteration method via modified Riemann-Liouville derivative. J. King. Saud. Univ. (Science). 2011;413-417.

[45] Merdan M, Analytical approximate solutions of fractionel convection-diffusion equation with modified Riemann-Liouville derivative by means of fractional variational iteration method. Iranian Journal of Science & Technology, IJST. 2013;A1:83-92.

[46] Dehghan M, Shakeri, F. Solution of an integro-differential equation arising in oscillating magnetic fields using he's homotopy perturbation method. Prog. Electromagn. Res. 2008;78:361-376.

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