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Improved Power System State Estimation by Selected Node Technique

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Abstract

The state estimator is integral part of any Energy Management System. First and foremost the state estimation must be executed followed by system control, tie-line control, economic dispatch, security analysis etc. Most importantly the system voltage controls and the tie-line power controls must be handled within milliseconds to a few seconds. Obviously, in order to meet the requirements, state estimator should be able to process the results very fast. Due to the kind of complexity associated with the power system it's very difficult to carry out the estimation in very short time. The author, H.N. Udupa & Dr. H.R. Kamath [1,2] had suggested a new innovative method to solve this complex problem in desired time without compromising on the results accuracy. In the said new approach, State Estimations are computed at each Node level.

This paper presents a unique technique to carried-out the State Estimation at selected Node Areas instead of every Node Area. As the network is interconnected, by selecting suitable Node Area it is possible to estimate all the state variables of the system. The method of selecting the Node Area is detailed in this paper. A node/bus along with its connected nodes/buses is called "Node Area". By computing the SE only at Selected Nodes reduces the complexity of the system and also results in huge cost saving. The Node Area level of state

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estimation technique is suitable for smart grid application. This paper presents the Node Area selection technique along with its computational time and comparison with the conventional Integrated State Estimation (ISE) and Node Level State estimation.

Keywords: SE- State Estimation, WLS – Weight Least Square, NR – Newton-Raphson, ISE – Integrated State estimation, NASE – Node Area State estimation, SNSE – Selected Node State estimation, NA – Node Area- A node along with its connected Node is referred as Node Area.

Symbols

*J – Jacobian matrix of the order of (m * (2n-1))*
H_{ij} *– Sub matrices of Jacobian matrix 'J'*
m – Total number of network measurements taken
n - Total number of network bus/node.
*W – Diagonal Weight matrix of the order of (m * m)*
$[x]^T = [\delta_1, \delta_2, ...\delta_{n-1}; v_1, v_2, ...v_n,]$*, state variables*
$[P_i, Q_i]$ *– Injected real and reactive powers respectively*
$[p_{ij}, q_{ij}]$ *- Real and reactive line flow respectively*
$[V_i, \delta_i]$ *– Voltage and angle respectively.*
$\Delta z = z^{measured} - z^{calculated}$
$A = (J^T * W * J)$ *of the order of (2n-1)*(2n-1);* $b = (J^T * W * \Delta z))$ *of the order of (2n-1)*1*

1 Introduction

The NR method of State Estimation is commonly applied in electric power system State Estimation. The N R method is also used in Two Level State Estimation approach. In Two Level State Estimation approach, a large power system network is physically divided into smaller area and initially, first level of Estimation is carried out at each area. At second level, boundary node state variables are estimated considering the first level results. In this regard various research contributions are available in the literature. A few such literatures are by M. Y. Patel and A. A. Girgis [3], Durgaprasad, S. S. Thakur, [4], Bahgat A. , Sakr M. M. F., El-Shafei A. R., [5], K.L. Mofreh, M. Salem, McColl, R.D.Moffatt, and SULLEY J.L [6]. Habiballah. [7], Bahgat A. , Sakr M. M. F., El-Shafei A. R., [8], Van Cutsem, T.H., Horward, J.L., and Ribben-Pavella M. [9]. A few other techniques used by the researchers are "Power System Tracking State Estimator" by W. W. Kotiuga, [10], "Real-time state estimation" by Kurzyn, M.S. [11] and "Hierarchical state estimation" by Van Cutsem, T.H. Horward J.L., Ribbens-Pavella, M., and EL-Fattah,Y.M, [12].

In case of NASE, neither the given network is physically divided nor is any approximation made in NR solution technique [2]. Even though NASE technique is unique and innovative but on the other side, every Node requires the state of the art computing and communication system. This in turn will results in increase in overall computing cost. This paper presents the SNSE technique which takes care of the cost factor.

1.1 Multi-processing [1]

Referring to Appendix-1, the standard NR solution for ISE is as follows

Let $A = \left(J^T * W * J\right)$ & $b = \left(J^T * W * \Delta z\right)$

$$\text{Then,} \quad A * \Delta x = b \tag{1}$$

We must view equation (1) as a prescription for an iterative procedure which in finite number of steps will compute the state vector 'x' to a certain degree of accuracy. Hence, vector 'x' should therefore be changed accordingly until the convergence is reached.

$$\begin{aligned} x^{(c+1)} &= x^{(c)} + (J^T * W * J)^{-1} * (J^T * W * \Delta z) \\ &= x^{(c)} + A^{-1} * b \\ &= x^{(c)} + \Delta x^{(c)} \end{aligned} \tag{2}$$

$$J_j^T * W_{jj} * J_j = A_j \& J_j^T * W_{jj} * \Delta z_j = b_j \ldots\ldots\ldots\ldots \tag{3}$$

Where, j = 1, 2, 3, ., ., m;

$$\text{Where } A = [A_1 + A_2 + A_j + \ldots + A_m] \quad \ldots\ldots\ldots\ldots \tag{4}$$

$$b = [b_1 + b_2 + b_j + \ldots + b_m] \quad \ldots\ldots\ldots\ldots \tag{5}$$

For n[th] Node Area measurements Jacobian relation is as follows [1]

$$\begin{bmatrix} \Delta P_n \\ \Delta Q_n \\ \Delta p_{ij}^n \\ \Delta q_{ij}^n \\ \Delta v^n \\ \Delta \partial^n \end{bmatrix}_{\Delta z_{NAn}} = \begin{bmatrix} |H_1^n| |H_2^n| \\ |H_3^n| |H_4^n| \\ |H_5^n| |H_6^n| \\ |H_7^n| |H_8^n| \\ |H_9^n| |H_{10}^n| \\ |H_{11}^n| |H_{12}^n| \end{bmatrix}_{J_{NAn}} \begin{bmatrix} \Delta \delta_i \\ \Delta v_i \end{bmatrix}_{(\Delta x_i)_{NAn}} \ldots\ldots\ldots \tag{6}$$

Δp_{ij}^r, Δq_{ij}^r is the real and reactive line flow residuals ($p_{ij}^{r(measured)} - p_{ij}^{r(calculated)} = \Delta p_{ij}^r$) between the connected nodes of r[th] node area and similarly, $\Delta v^r, \Delta \partial^r$ is the voltage and angle residuals between the connected nodes of r[th] node area.

$(\ )_{NAi}$ The subscript 'NAi' refers to 'i[th]' node area

$$\sum_{j=1}^{n} (A_{NAj}) \Delta x = \sum_{j=1}^{n} (b_{NAj}) \tag{7}$$

1.2 Node Area State Estimation (NASE) [2]

As given in the NASE technique by H.N. Udupa, Dr. H.R. Kamath [2] Node Area State Estimation is as follows.

The kth Node area measurements many include $[P_k, Q_{k,} p_{ij}^k, q_{ij}^k, v^k, \partial^k]$. If sufficient measurements are made available at each node area, form equation (6) and (7) it can be written as

$$(J_{NAk}^T * W_{NAk} * J_{NAk})(\Delta x_i)_{NAk} = (J_{NAk}^T * W_{NAk} * z_{NAk})$$
$$A_{NAk}(\Delta x_i)_{NAk} = b_{NAk} \qquad (8)$$

where 'N_{Ak}' refers to k^{th} node area and $(\Delta xi)N_{Ak}$ is the state vector corresponds to k^{th} node area. $(x_i^{c+1})_{NAk} = (x_i^c)_{NAk} + \left(\Delta x_i^c\right)_{NAk}$; Where 'c' is the iteration count and 'k' =(1, 2, ., ., ., n). The equation (1) is for whole network whereas the eq (8) is for k^{th} node area.

2 Selected Node State Estimation (SNSE) Technique

2.1 SNSE Introduction

The Selected Node State Estimation (SNSE) technique provides a method to carry-out the Node Level State Estimation only at selected nodes of the whole network. The NASE only at certain selected Node Areas of the power system network is termed as 'Selected Node State Estimation'. In the Node Level State Estimation [2], State Estimation is carried-out at each Node Area independently. As the bus/Nodes are interconnected it is not necessary to compute the state variables at all the Nodes. For example, if Node- k is connected to Node-j, then the state variables (vk, vj, ∂k, ∂j) will be estimated at k^{th} Node Area and also at j^{th} Node Area. Hence, it is possible to estimate all the state variables of the system by computing the state variables only at a few selected Nodes. The Nodes are selected to meet the following criteria,

i. The combination of all the connected Nodes of the Selected Node Areas should cover all the Nodes of the system/network.
ii. There has to be connectivity between selected Node Areas either directly or through a common Node so that a Node path can be established from NA-1 to NA-n.

2.2 Node Area Selection Steps

Step1:- Start from node having maximum number of connected Nodes: - NA-i(max) List all the Nodes of NA-i(max).
Step2:- Among the connected nodes of NA under consideration identify which node has more number of parallel paths (more number of connected nodes).
Step3: -Now consider the new NA identified from step 2.
Step4:- Identify the new Nodes (other than covered so far) and add to the Node list
Step5:- Check whether noted node list consists of all nodes of the given system?
Step6: -If Yes stop, else go to step 2.

(Note: - there may a case wherein equal number parallel path among two or more than two connected nodes, in such case need to go one more level deeper).

2.3 Assumptions

- All measurements are taken at the same instant
- At selected nodes sufficient numbers of measurements are taken or in other words, at each selected node total of injected power measurements + voltage measurements + the line flow measurements should be equal to or greater than the number of state variables of that node Area.
- The nodes are selected in such a way that by carrying out SE for these nodes complete system state variables covered. Measurement redundancy may also be provided to ensure sufficient measurements at selected Node Areas.

2.4 Example: -13Bus System

2.4.1 Node selection steps: - 13 Bus example

The Node Area logic given in the section II part B is demonstrated below for 13 bus graph shown in Fig. 1.

Table 1. Node selection table

NA	Connected nodes	Self + connected nodes	Selection sequence	New node case1	New node case2
1	2	1,2		0	0
2	1,3	2,1,3	4(stop)	1,2	1,2,3
3	2,4	3,2,4		0	0
4	3,5,8	4,3,5,8	3	3	0
5	4,6,9	5,4,6,9	2	4,6	4,6
6	5,7	6, 5,7		0	0
7	6,10	7, 6,10		0	0
8	4,9	8, 4,9		0	0
9	5,8,10,12	9,5,8,10,12	1(start)	5,8,9, 10,12*	5,8,9, 10,12*
10	7,9,11	10,7,9,11	2	7,11	7,11
11	10,13	11,10,13	3	13	13
12	9,13	12, 9,13		0	0
13	11,12	13, 11,12		0	0

* Start from Node-9. The node selection results are tabulated in Table-1: - Node Selection table

- Node Area-9* is having maximum number of connected nodes = 4
  - At NA-9, Nodes covered are -5,8, 9,10,12;
- Number of connected nodes at NA-10 & NA-5 = 3
  - At NA-10, new Nodes covered are – 7, 11; (other than previous NA Nodes);
- Number of connected nodes at NA-7 & NA-11= 2, but Node-7 has already been covered.
  - At NA-11, new Node covered is – 13; (other than previous NA Nodes);
  - At NA-5, new Nodes covered are – 4, 6; (other than previous NA Nodes);

- Number of connected nodes at NA-4 & NA-6= 2, but Node 6 is connected to node-5 and Node-7 which are listed already.
  - At NA-4, new Node covered is – 3; (other than previous NA Nodes);
- At NA-3, new Nodes covered are – 2;

Case-1

- At NA-2, new Node covered is – 1
  Node-3 is common between NA-2 and NA- 4, hence NA-3 can be dropped.
- Therefore, at NA-2, new Nodes covered are – 1, 2;
- Numbers of Nodes listed so far are –1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13; all nodes are listed – stop.

Case-2

- If NA-4 is dropped, at NA-2, new Nodes – 1,2,3

2.4.2 Case-1

The Figs. 2 and 3 represents the Selected Node Area and node connectivity diagram. The dotted link between node-2 and node-4 in Fig. 3, shows that Node-3 is common between NA4 and NA2. The Case-1 satisfies the criteria stated in section 2.1.

2.4.3 Case-2

The Figs. 4 and 5 represents the Selected Node Area and node connectivity diagram for case-2. The dotted link between node-2, 3, 4 and node-5 in Fig. 5, shows that there is no common Node between NA2 and NA5. The Case-2 does not satisfy the criteria stated in section 2.1. Hence case-2 is not preferable.

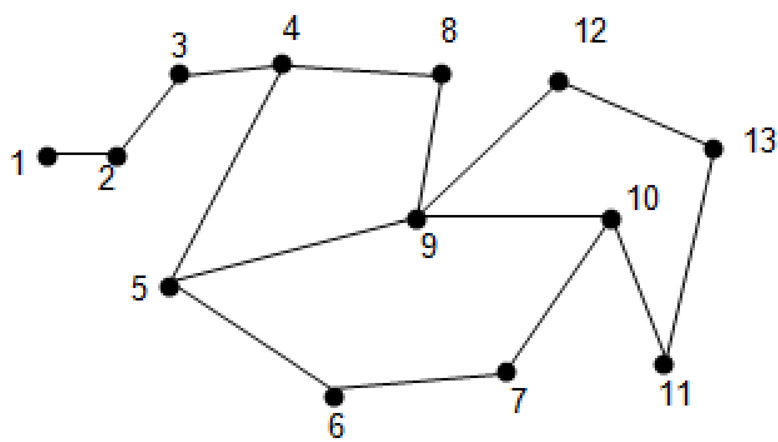

Fig. 1. 13 bus test system

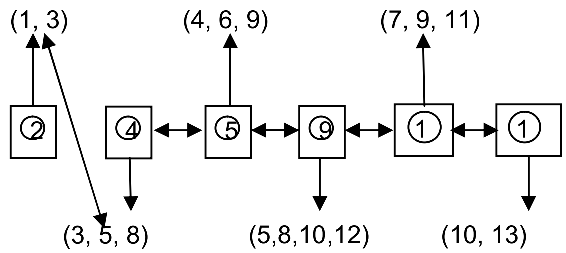

Fig. 2. Case-1 selected node area connectivity

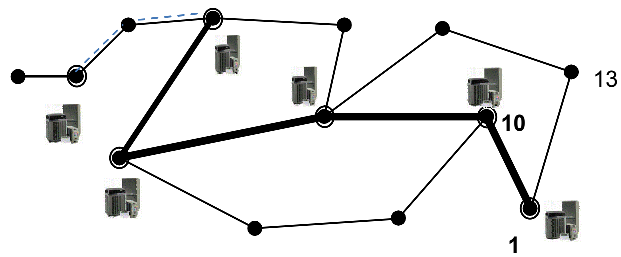

Fig. 3. Case-1 selected node connectivity

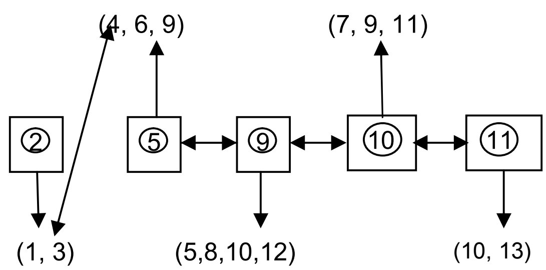

Fig. 4. Case-2 selected node area connectivity

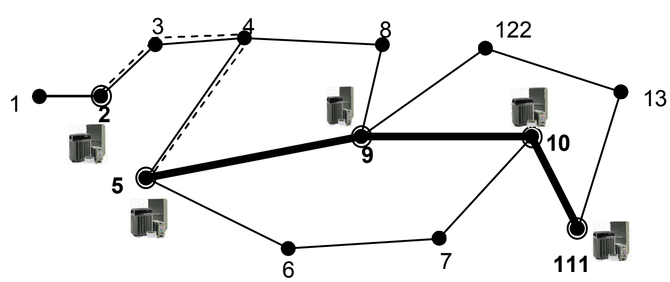

Fig. 5. Case-2 selected node connectivity

3 Example and Results

The above concept is tested on a 13 and 30 bus test systems for ISE and SNSE. The estimation results of both the methods are found to be same up to five decimal. The results of computational time are tabulated in the following section. In both the methods (ISE and SNSE) proper indexing is used to avoid non-zero computations.

3.1 Input Data: - Input Data for 13 and 30 Bus Systems are given in Appendix-2

3.2 Results

The 13 bus and 30 bus ISE and SNSE results are tabulated in the Tables 2 and 3 respectively. The ISE and SNSE voltage comparison for 13 bus, 30 bus system is shown in chart 1, chart 3 and phase angle comparison is shown in chart 2, chart 4 respectively.

Table 2. ISE& SNSE results (13 bus test system) number of iteration = 3

Bus no.	V-ISE	∂-ISE	V-SNSE	∂-SNSE
1	1.05328	0	1.05327	0
2	0.979382	-0.0605903	0.979365	-0.060592
3	0.958245	-0.0861526	0.958230	-0.086152
4	0.945926	-0.101909	0.945931	-0.101910
5	0.928904	-0.123005	0.928905	-0.123004
6	0.925919	-0.126983	0.925939	-0.127
7	0.924528	-0.128747	0.924536	-0.12875
8	0.926483	-0.1247	0.926472	-0.1247
9	0.925293	-0.127228	0.925299	-0.12723
10	0.923413	-0.130107	0.923408	-0.13011
11	0.919752	-0.135169	0.919748	-0.13517
12	0.922148	-0.131696	0.9221141	-0.1317
13	0.920722	-0.133764	0.920714	-0.13377

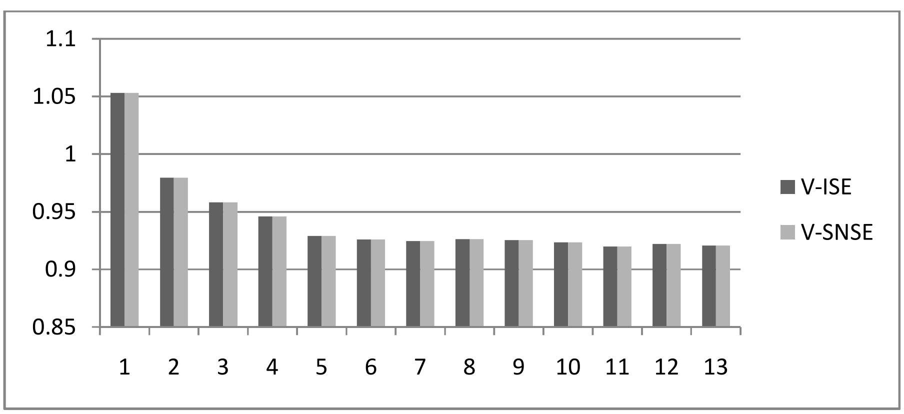

Chart 1. ISE Vs SNSE voltage profile for 13 bus test system

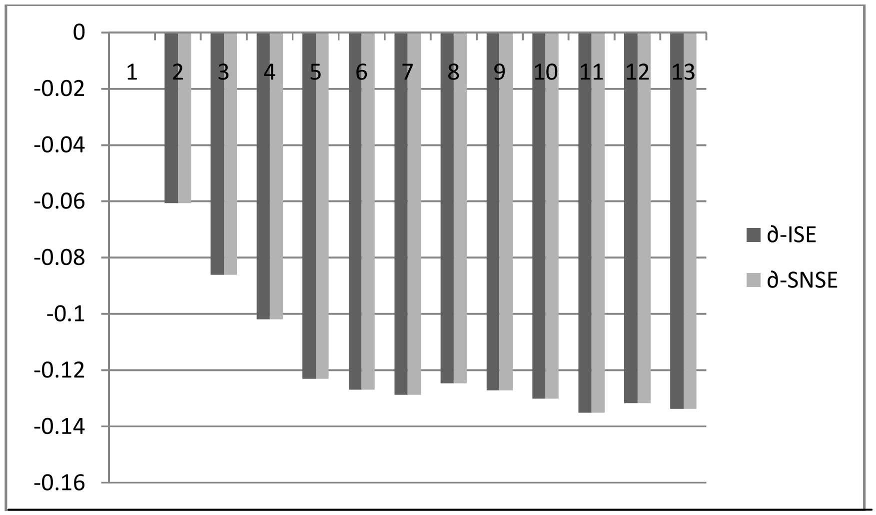

Chart 2. ISE Vs SNSE phase angle profile for 13 bus test system

Table 3. ISE & SNSE results (30 bus test system) number of iteration = 3

Bus	V-ISE	δ-ISE	V-SNSE	δ-SNSE
1	1.05	0	1.05	0
2	1.0338	-0.04773	1.0338	-0.04773
3	1.0313	-0.08174	1.0313	-0.08174
4	1.0263	-0.09791	1.0263	-0.09791
5	1.0058	-0.15702	1.0058	-0.15702
6	1.0208	-0.1127	1.0208	-0.1127
7	1.0069	-0.14011	1.0069	-0.14011
8	1.023	-0.11303	1.023	-0.11303
9	1.0332	-0.14021	1.0332	-0.14021
10	1.0183	-0.17333	1.0183	-0.17333
11	1.0913	-0.10711	1.0913	-0.10711
12	1.0399	-0.16419	1.0399	-0.16419
13	1.0883	-0.14326	1.0883	-0.14326
14	1.0236	-0.17999	1.0236	-0.17999
15	1.0179	-0.18089	1.0179	-0.18089
16	1.0235	-0.17291	1.0235	-0.17291
17	1.0144	-0.17697	1.0144	-0.17697
18	1.0057	-0.19076	1.0057	-0.19076
19	1.0017	-0.19314	1.0017	-0.19314
20	1.0051	-0.18911	1.0051	-0.18911
21	1.0061	-0.18167	1.0061	-0.18167
22	1.0069	-0.18148	1.0069	-0.18148
23	1.0053	-0.18721	1.0053	-0.18721
24	0.9971	-0.18938	0.997099	-0.18938
25	1.0086	-0.19045	1.0086	-0.19045

Bus	V-ISE	δ-ISE	V-SNSE	δ-SNSE
26	0.990798	-0.1979	0.990799	-0.1979
27	1.0245	-0.18617	1.0245	-0.18617
28	1.0156	-0.1199	1.0156	-0.1199
29	1.0047	-0.20759	1.0047	-0.20759
30	0.993197	-0.22297	0.993201	-0.22297

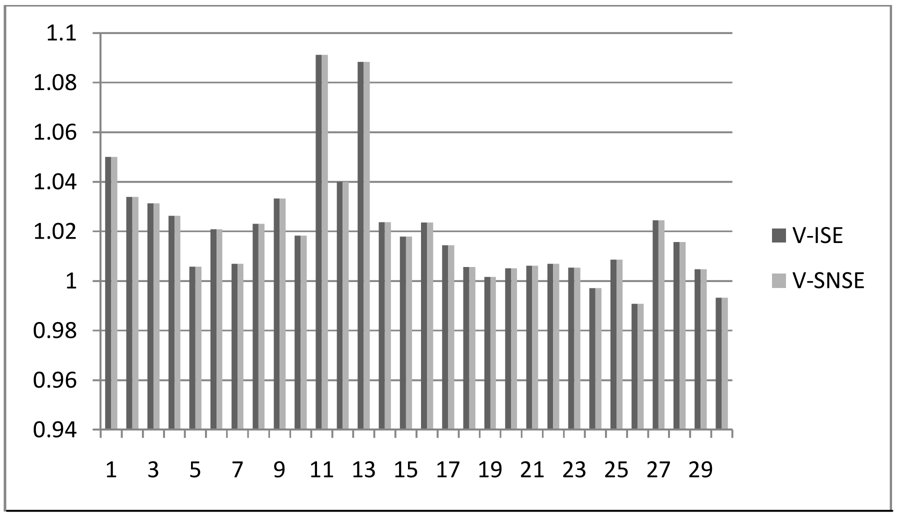

Chart 3. ISE Vs SNSE voltage profile for 30 bus test system

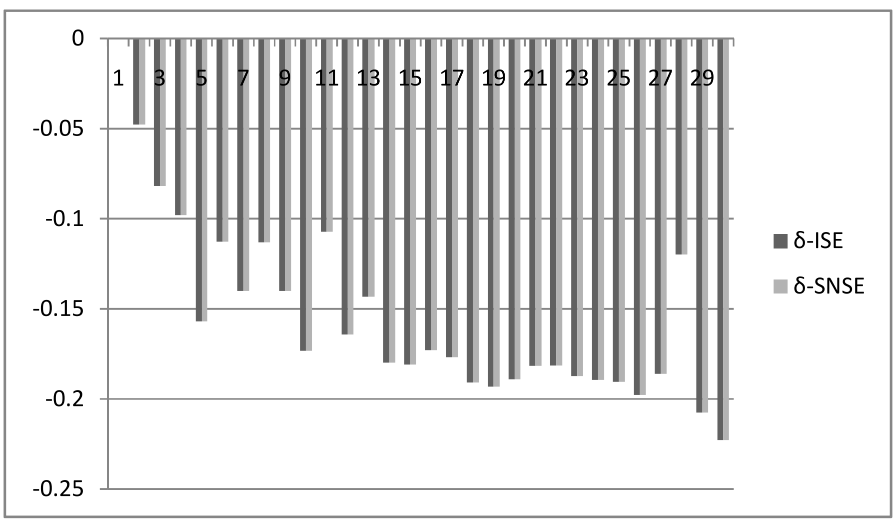

Chart 4. ISEVs SNSE Angle profile for 30 bus test system

Table 4. Computational time for 13 and 30 bus systems

Type	Time (ms)
13 bus- SNSE	0.19 at NA9
30 bus-SNSE	0.41 at NA6
13 bus- ISE	2.16
30 bus-ISE	27.60

Note: -For 30 bus system the Node Area selected are

Node area	NA 6	NA1 0	NA1 2	NA1 5	NA2 7	NA 4	NA2 5	NA1 9	NA 5	NA 1	NA11
No. of connected nodes	7	6	5	4	4	4	3	2	2	2	1

- From Table 4, for 13 bus $(SNSE)_{max}$ computing time is observed at Node Area9 (NA9),because the node 9 has maximum connected nodes and for 30 bus $(SNSE)_{max}$ computing time is observed at Node Area 6 (NA6), because the node 6 has maximum connected nodes. The above timings are obtained using profiling tool. These timing are also dependent on the processor and the operating system. The Node-9 (NA9) of 13 bus system is has 4 connected nodes the time taken to compute is 0.19ms and the Node-6 (NA6) of 30 bus system is has 7 connected nodes the time taken to compute is 0.41ms.

4 Conclusion

As the power system network size increases, number node/bus also increase. But regardless of the size of the network, the size of the Node Area will depend only on its connected nodes. Hence, Node Area size does not depend on the network size (total number of node/bus in the network).Disadvantage of NASE technique [2] is the requirement of complete state of art data acquisition, communication and computation system at each node. And also other requirement is the increase in the measurements because at each node area sufficient measurements should be made available to obtain reliable results. The SNSE technique suggested here provides solution to the above said problems. Huge increase in the cost due to the said disadvantages of NASE can be avoided using SNSE technique. In case of SNSE same method (NASE) is applied only at selected Node Areas of the whole network. The results of SNSE is compared with the results of ISE (standard method) and found to be same up to five decimal. It is evident from the results that the SNSE is a good solution for large size, complex state estimation. The maximum computational time is the time taken by the Node Area having maximum number of connected nodes. In the above 13bus example, it is evident that NA9 is having maximum number of connected nodes and consumes maximum computing time which is equals to 0.19milli sec and for 30 bus, it is 0.41ms. Here, ISE time has no relevance because NASE/SNSE time depends on the Node Area having maximum number of connected nodes but not on the total network size. Practically, a node may have a maximum of 10 or 12 connected nodes. Hence, actual SE computational time required is the time taken by the Node Area- having 10 or 12 connected nodes. Using SNSE over NASE technique reduces the system cost.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX-1

1. NR STATE ESTIMATION

1.1 Basic Derivation

Let ‘z’ be the measurement vector as a function of the state vector ‘x’,

The state vector $x^T = [v_1, v_2, \ldots v_n, \delta_1, \delta_2, \ldots \delta_{n-1}]^T$; (1.1.1)

Where

- ‘n’ is the number of state variable,
- [$v_1, v_2, \ldots v_n$] are the respective voltage magnitudes and
- $\delta_1, \delta_2, \ldots \delta_{n-1}$ are the respective phase angles.
- The dimension of the state vector ‘x’ = (2n-1)*1, because one of the bus is considered as the phase angle reference bus.

Let

- ‘s’ be the Structure of the network
- ‘c’ be the information of the network

$$\text{Then } z = f(x, c, s) \quad (1.1.2)$$

The equation-2 computes ‘m’ measurement values as a function of state vector ‘x’

$$z = f(x) + e \quad (1.1.3)$$

Where ‘e’ is the ‘m’ dimensional error vector as all the measurements may have some errors.

$$\text{error vector } \quad e = z - f(x) \quad (1.1.4)$$

To estimate the state variables (x0) the norm of ‘e’ should be minimal.

Hence the cost function to estimate the sate variables

$$\text{'U'} = \|e\|^2 = e^2 * e = \sum_{i=1}^{m} [z_i - f_i(x)]^2 \quad (1.1.5)$$

This is also known as “Least Square Minimization (WLS)” technique.

The error vector includes nonlinear vector function f(x), in-order to estimate x, an initial value ‘x0’ is assumed and using taylor expression it can be written as

$$f(x) = f(x_0) + f'(x_0)\Delta x + f''(x_0)\frac{\Delta x}{2!} \quad (1.1.6)$$

Neglecting the higher order terms and using the matrices, we have

$$f(x) = f(x_0) + J(x_0)\Delta x$$
$$f = f_0 + J_0 \Delta x \quad (1.1.7)$$

The Jacobian matrix 'J' is defined as

$$\mathrm{J}_{\mathrm{k,n}} = \frac{\partial f_k}{\partial x_n} \quad (1.1.8)$$

$e = z - f$; let $\Delta z = z - f_0$ or $z = \Delta z + f_0$ (1.1.9)

Substituting the equations (7) and (9) in equation (5),

$$U(x) = e^T * e = (\Delta z + f_0 - f_0 - J_0 * \Delta x)^T * (\Delta z - J_0 * \Delta x)$$
$$= (\Delta z - J_0 * \Delta x)^T * (\Delta z - J_0 * \Delta x) \quad (1.1.10)$$

minimizing $\frac{\partial U(x)}{\partial x} = 0$, equation (3.1.10) reduces to

$$\left(J_0^T * J_0\right) * \Delta x = \left(J_0^T * \Delta z\right) \quad (1.1.11)$$

Let

- 'W' be the weight matrix, which is a diagonal matrix
- 'Wii' is the diagonal element, which is nothing but the Standard deviation (σ) of the meter.

Then $U(x) = e^T * \mathrm{W} * \mathrm{e}$ (1.1.12)

$$\left(J_0^T * W * J_0\right) * \Delta x = \left(J_0^T * W * \Delta z\right) \quad (1.13)$$

$let \;\; J_0 \Rightarrow J$

Let $A = \left(J^T * W * J\right)$ & $\mathrm{b} = \left(J^T * W * \Delta z\right)$ (1.1.14)

Then, $A * \Delta x = b$ (1.1.15)

We must view equation (3.1.15) as a prescription for an iterative procedure which in finite number of steps will compute the state vector 'x' to a certain degree of accuracy. Hence, vector 'x' should therefore be changed accordingly until the convergence is reached.

$$x^{(c+1)} = x^{(c)} + (\mathrm{J}^{\mathrm{T}} * \mathrm{W} * \mathrm{J})^{-1} * (\mathrm{J}^{\mathrm{T}} * \mathrm{W} * \Delta \mathrm{z})$$
$$= x^{(c)} + A^{-1} * b$$
$$= x^{(c)} + \Delta x^{(c)} \quad (1.1.16)$$

1.2 Jacobian Formation

$$P_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\cos\left(\delta_i - \delta_j - \theta_{ij}\right) \quad (1.2.1)$$

$$Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\sin\left(\delta_i - \delta_j - \theta_{ij}\right) \quad (1.2.2)$$

$$S_{ij} = V_i I^* = V_i (I_{ij} - I_{i0})^*; \text{ where, } \quad I_{ij} = (V_i\text{-}V_j) \;\;\&\;\; I_{i0} = V_i y_{i0} \quad (1.2.3)$$

Where

$V_i = |V_i|\angle\delta_i$; $V_i = |V_j|\angle\delta_j$; $y_{ij} = |y_{ij}|\angle\alpha_{ij}$; $y_{i0} = |y_{i0}|\angle\alpha_{i0}$

$y_{ij} \rightarrow$ Primitive element, line admittance between ith and jth bus

$y_{i0} \rightarrow$ Shunt admittance from corresponding (ith) bus to ground

p_{ij} and q_{ij} $\rightarrow$ Real and Reactive power flow between ith jth bus respectively

Simplifying the equation (3.2.3), yields to

$$S_{ij} = |V_i|^2 |y_{ij}|\angle-\alpha_{ij} - |V_i||V_j||y_{ij}|\angle(\delta_i - \delta_j - \alpha_{ij}) + |V_i|^2 |y_{i0}|\angle-\alpha_{i0} \quad (1.2.4)$$

$$p_{ij} = |V_i|^2 |y_{ij}|\cos(-\alpha_{ij}) - |V_i||V_j||y_{ij}|\cos(\delta_i - \delta_j - \alpha_{ij}) + |V_i|^2 |y_{i0}|\cos(-\alpha_{i0}) \quad (1.2.5)$$

$$q_{ij} = |V_i|^2 |y_{ij}|\sin(-\alpha_{ij}) - |V_i||V_j||y_{ij}|\sin(\delta_i - \delta_j - \alpha_{ij}) + |V_i|^2 |y_{i0}|\sin(-\alpha_{i0}) \quad (1.2.6)$$

$$\begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta p_{ij} \\ \Delta q_{ij} \\ \Delta v_i \\ \Delta\delta_i \end{bmatrix} = \begin{bmatrix} |H_1||H_2| \\ |\mathrm{H}_3||\mathrm{H}_4| \\ |\mathrm{H}_5||\mathrm{H}_6| \\ |\mathrm{H}_7||\mathrm{H}_8| \\ |\mathrm{H}_9||\mathrm{H}_{10}| \\ |\mathrm{H}_{11}||\mathrm{H}_{12}| \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta v_i \end{bmatrix} \quad (1.2.7) \qquad H_1 = \begin{bmatrix} \frac{\partial P_1}{\partial\delta_1} \cdots\cdots \frac{\partial P_1}{\partial\delta_{n-1}} \\ . \\ . \\ \frac{\partial P_n}{\partial\delta_1} \cdots\cdots \frac{\partial P_n}{\partial\delta_{n-1}} \end{bmatrix} = \frac{\partial \mathrm{P_i}}{\partial\delta_j} \quad (1.2.8)$$

;

Note : - $\mathrm{H}_{\text{odd number}}$ - Number of coloum is (n - 1) because *o*ne bus is considered as angle reference bus

$$H_{1(ii)} = -|V_i^2||Y_{ii}|\sin\theta_{ii} - Q_i^{cal} = \frac{\partial P_i}{\partial\delta_i} \quad (1.2.9)$$

$$\underset{j!=i}{H_{1(ij)}} = |V_i||V_j||Y_{ij}|\sin\left(\delta_i - \delta_j - \theta_{ij}\right) = \frac{\partial P_i}{\partial\delta_j} \quad (1.2.10)$$

$$H_{2(ii)} = \left(|V_i^2||Y_{ii}|\cos\theta_{ii} + P_i^{cal}\right)\big/|V_i| = \frac{\partial P_i}{\partial|V_i|} \quad (1.2.11)$$

$$\underset{j!=i}{H_{2(ij)}} = |V_i||Y_{ij}|\cos(\delta_i - \delta_j - \theta_{ij}) = \frac{\partial P_i}{\partial|V_j|}; \quad (1.2.12)$$

$$H_{3(ii)} = P_i^{cal} - |V_i^2||Y_{ii}|\cos\theta_{ii} = \frac{\partial Q_i}{\partial \delta_i} \quad (1.2.13)$$

$$\underset{j!=i}{H_{3(ij)}} = -|V_i||V_j||Y_{ij}|\cos(\delta_i - \delta_j - \theta_{ij}) = \frac{\partial Q_i}{\partial \delta_j} \quad (1.2.14)$$

$$H_{4(ii)} = \left(Q_i^{cal} - |V_i^2||Y_{ii}|\sin\theta_{ii}\right) / |V_i| = \frac{\partial Q_i}{\partial |V_i|} \quad (1.2.15)$$

$$\underset{j!=i}{H_{4(ij)}} = |V_i||Y_{ij}|\sin(\delta_i - \delta_j - \theta_{ij}) = \frac{\partial Q_i}{\partial |V_j|}; \quad (1.2.16)$$

$$H_5 = \frac{\partial p_{ij}}{\partial \delta_i} = |V_i||V_j||y_{ij}|\sin(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'i'} \quad (1.2.16)$$

$$= \frac{\partial p_{ij}}{\partial \delta_j} = -|V_i||V_j||y_{ij}|\sin(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'j'} \quad (1.2.17)$$

$$H_6 = \frac{\partial p_{ij}}{\partial V_i} = 2|V_i||y_{ij}|\cos(-\alpha_{ij}) - |V_j||y_{ij}|\cos(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'i'} \quad (1.2.18)$$

$$= \frac{\partial p_{ij}}{\partial V_j} = -|V_i||y_{ij}|\cos(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'j'} \quad (1.2.19)$$

$$H_7 = \frac{\partial q_{ij}}{\partial \delta_i} = -|V_i||V_j||y_{ij}|\cos(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'i'} \quad (1.2.20)$$

$$= \frac{\partial q_{ij}}{\partial \delta_j} = |V_i||V_j||y_{ij}|\cos(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'j'} \quad (1.2.21)$$

$$H_8 = \frac{\partial q_{ij}}{\partial V_i} = 2|V_i||y_{ij}|\sin(-\alpha_{ij}) - |V_j||y_{ij}|\sin(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'i'} \quad (1.2.22)$$

$$= \frac{\partial q_{ij}}{\partial V_j} = -|V_i||y_{ij}|\sin(\delta_i - \delta_j - \alpha_{ij}) \text{ in column 'j'} \quad (1.2.23)$$

$$H_9 \,\&\, H_{12} \rightarrow \text{all elements of the matrix are zero} \quad (1.2.24)$$

$$H_{10} \,\&\, H_{11} \rightarrow \text{both are unity diagonal matrix} \quad (1.2.25)$$

APPENDIX-2

2.1 Input data: - Input data for 13 and 30 bus systems

TABLE 2.1.1 -LINE DATA FOR 13 AND 30 BUS SYSTEMS

Node-i	Node-j	r	x	i	j	r	x	y/2
1	2	0.00148	0.0028676	1	2	0.0192	0.0575	0.0264
2	3	0.000438	0.00124174	1	3	0.0452	0.1852	0.0204
3	4	0.000277	0.00078488	2	4	0.057	0.1737	0.0184
4	5	0.000598	0.00166769	3	4	0.0132	0.0379	0.0042
4	8	0.0016	0.00310017	2	5	0.0472	0.1983	0.0209
5	6	0.000343	0.0009719	2	6	0.0581	0.1763	0.0187
5	9	0.000343	0.0009719	4	6	0.0119	0.0414	0.0045
6	7	0.000324	0.00091669	5	7	0.046	0.116	0.0102
7	10	0.000324	0.00091669	6	7	0.0267	0.082	0.0085
8	9	0.000294	0.0008338	6	8	0.012	0.042	0.0045
9	10	0.000532	0.00150562	6	9	0	0.208	0
9	12	0.000378	0.0010705	6	10	0	0.556	0
10	11	0.000588	0.00166488	9	11	0	0.208	0
11	13	0.000324	0.00091669	9	10	0	0.11	0
12	13	0.000368	0.00104355	4	12	0	0.256	0
--	--	--	--	12	13	0	0.14	0
--	--	--	--	12	14	0.1231	0.2559	0
--	--	--	--	12	15	0.0662	0.1304	0
--	--	--	--	12	16	0.0945	0.1987	0
--	--	--	--	14	15	0.221	0.1997	0
--	--	--	--	16	17	0.0825	0.1932	0
--	--	--	--	15	18	0.107	0.2185	0
--	--	--	--	18	19	0.0639	0.1292	0
--	--	--	--	19	20	0.034	0.068	0
--	--	--	--	10	20	0.0936	0.209	0
--	--	--	--	10	17	0.0324	0.0845	0
--	--	--	--	10	21	0.0348	0.0749	0
--	--	--	--	10	22	0.0727	0.1499	0
--	--	--	--	21	22	0.0116	0.0236	0
--	--	--	--	15	23	0.1	0.202	0
--	--	--	--	22	24	0.115	0.179	0
--	--	--	--	23	24	0.132	0.27	0
--	--	--	--	24	25	0.1885	0.3292	0
--	--	--	--	25	26	0.2544	0.38	0
--	--	--	--	25	27	0.1093	0.2087	0
--	--	--	--	28	27	0	0.396	0
--	--	--	--	27	29	0.2198	0.4153	0
--	--	--	--	27	30	0.3202	0.6027	0
--	--	--	--	29	30	0.2399	0.4533	0
--	--	--	--	8	28	0.0636	0.2	0.0214
--	--	--	--	6	28	0.0169	0.0599	0.0065

TABLE 2.1.2 - VOLTAGE AND ANGLE MEASUREMENTS FOR 13 AND 30 BUS SYSTEM

13 bus system			30 bus system		
Bus No.	**Vi**	**∂i (rad)**	**Bus No.**	**Vi**	**∂i (rad)**
1	1.053269	0	1	1.05	--
3	0.958231	-0.08616	2	1.0338	-0.04773
5	0.928889	-0.12301	3	1.0313	-0.08174
9	0.925278	-0.12723	6	1.0208	-0.1127
10	0.923398	-0.13011	7	1.0069	-0.14011
11	0.919737	-0.13517	8	1.023	-0.11303
13	0.920707	-0.13377	9	1.0332	-0.14021
--	--	--	11	1.0913	-0.10711
--	--	--	12	1.0399	-0.16419
--	--	--	13	1.0883	-0.14326
--	--	--	14	1.0236	-0.17999
--	--	--	17	1.0144	-0.17697
--	--	--	19	1.0017	-0.19314
--	--	--	20	1.0051	-0.18911
--	--	--	21	1.0061	-0.18167
--	--	--	22	1.0069	-0.18148
--	--	--	23	1.0053	-0.18721
--	--	--	24	0.9971	-0.18938
--	--	--	25	1.0086	-0.19045
--	--	--	26	0.9908	-0.1979
--	--	--	30	0.9932	-0.22297

TABLE 2.1.3: -INJECTED POWER MEASUREMENTS FOR 13 AND 30 BUS SYSTEMS

13BUS System			30 BUS System	
Bus No.	**Pi**	**Qi**	**Pi**	**Qi**
1	28.162	13.18	1.38525	-0.0281959
2	-5.89546	-1.07074	0.358726	-0.102251
3	-3.42084	0.330078	-0.0239129	-0.0117302
4	-2.47793	0.706497	-0.0759117	-0.0154608
5	-6.9646	-0.50879	-0.696608	0.0359962
6	-4.15897	0.041931	-0.000182518	-0.000893709
7	-2.5218	0.616272	-0.228155	-0.109163
8	-6.13422	-0.9848	0.0500946	0.0487403
9	-4.89722	0.894775	1.22E-05	0.000443043
10	-2.59491	0.689423	-0.0580753	-0.217182
11	-5.998	-0.86444	0.179375	0.307797
12	-3.93211	-0.04755	-0.111825	-0.0740924
13	-2.38309	0.562073	0.169178	0.378012
14	--	--	-0.0620029	-0.016046
15	--	--	-0.0823402	-0.0255536
16	--	--	-0.0348603	-0.0176984
17	--	--	-0.0901929	-0.0583336
18	--	--	-0.0321083	-0.00917765
19	--	--	-0.0948698	-0.0336671
20	--	--	-0.020027	-0.00706335
21	--	--	-0.176513	-0.114945
22	--	--	0.00164309	0.00324485

13BUS System			30 BUS System	
Bus No.	Pi	Qi	Pi	Qi
23	--	--	-0.0320577	-0.0160815
24	--	--	-0.0871662	-0.106997
25	--	--	-2.02E-05	-1.78E-05
26	--	--	-0.0349631	-0.0229311
27	--	--	-2.02E-05	2.31E-05
28	--	--	-1.66E-05	-7.13E-05
29	--	--	-0.0239568	-0.00890309
30	--	--	-0.106086	-0.0190879

TABLE 2.1.4: -LINE FLOW MEASUREMENT FOR 13 AND 30 BUS SYSTEMS

13BUS System					30BUS System				
NA	Node -i	Node - j	pij	qij	NA	Node-i	Node-j	p_{ij}	q_{ij}
1	1	2	28.5327	13.0812	1	1	2	0.905766	-0.0142242
2	1	2	28.5327	13.0812	1	1	3	0.479483	-0.0139709
2	2	3	22.4874	8.978	2	1	2	0.905766	-0.0142242
3	2	3	22.4874	8.978	2	2	4	0.292082	-0.0631865
3	3	4	20.9456	7.7882	2	2	5	0.581169	0.0165859
4	3	4	20.9456	7.7882	2	2	6	0.376949	-0.0553518
4	5	4	-12.823	-4.76946	3	1	3	0.479483	-0.0139709
4	4	8	7.53516	2.11797	3	3	4	0.446142	-2.01E-02
5	5	4	-12.823	-4.76946	4	2	4	0.292082	-0.0631865
5	5	6	4.02863	1.43748	4	3	4	0.446142	-0.0201448
5	5	9	4.40723	1.90301	4	4	6	0.382601	0.024396
6	6	5	-4.02136	-1.41695	4	4	12	0.276134	-0.0451944
6	6	7	1.90622	0.733363	5	2	5	0.581169	0.0165859
7	6	7	1.90622	0.733363	5	5	7	-0.130421	0.0331114
7	7	10	1.4794	0.60305	6	2	6	0.376949	-0.0553518
8	8	4	-7.42565	-1.90575	6	4	6	0.382601	0.024396
8	8	9	2.72708	0.362727	6	6	7	0.3629	0.0507249
9	9	5	-4.39807	-1.87707	6	6	8	-0.0066594	-0.0562558
9	9	8	-2.72452	-0.35546	6	6	9	0.139488	-0.0588494
9	9	10	1.81613	0.516504	6	6	10	0.113266	0.00809595
9	9	12	4.02228	1.30623	6	6	28	0.138712	0.0431573
10	10	7	-1.47842	-0.60032	7	5	7	-0.130421	0.0331114
10	10	9	-1.8139	-0.51024	7	6	7	0.3629	0.0507249
10	10	11	2.93549	1.00072	8	6	8	-0.0066594	-5.63E-02
11	11	10	-2.92885	-0.98192	8	8	28	0.043405	0.00177545
11	11	13	-1.45929	-0.45715	9	6	9	0.139488	-5.88E-02
12	12	9	-4.01439	-1.2839	9	9	11	-0.179195	-2.86E-01
12	12	13	1.89218	0.59386	9	9	10	0.316618	0.145397
13	13	11	1.46017	0.459604	10	6	10	0.113266	0.00809595
13	12	13	1.89218	0.59386	10	9	10	0.316618	0.145397
--	--	--	--	--	10	10	20	0.0886135	0.0252389
--	--	--	--	--	10	10	17	0.0545927	0.0261477
--	--	--	--	--	10	10	21	0.157434	0.0931937
--	--	--	--	--	10	10	22	0.0756292	0.0409909
--	--	--	--	--	11	9	11	-0.179195	-0.285745
--	--	--	--	--	12	4	12	0.276134	-0.0451944
--	--	--	--	--	12	12	13	-0.168952	-0.357845

13BUS System					30BUS System				
NA	**Node -i**	**Node - j**	**pij**	**qij**	**NA**	**Node-i**	**Node-j**	**p_{ij}**	**q_{ij}**
--	--	--	--	--	12	12	14	0.0794531	0.0285367
--	--	--	--	--	12	12	15	0.179044	0.0856797
--	--	--	--	--	12	12	16	0.0714446	0.0520546
--	--	--	--	--	13	12	13	-0.168952	-0.357845
--	--	--	--	--	14	12	14	0.0794531	0.0285367
--	--	--	--	--	14	14	15	0.0166392	0.0108046
--	--	--	--	--	15	12	15	0.179044	0.0856797
--	--	--	--	--	15	14	15	0.0166392	0.0108046
--	--	--	--	--	15	15	18	0.0598374	0.0277604
--	--	--	--	--	15	15	23	0.0510092	0.0383417
--	--	--	--	--	16	12	16	0.0714446	0.0520546
--	--	--	--	--	16	16	17	0.0359018	0.0329218
--	--	--	--	--	17	16	17	0.0359018	0.0329218
--	--	--	--	--	17	10	17	0.0545927	0.0261477
--	--	--	--	--	18	15	18	0.0598374	0.0277604
--	--	--	--	--	18	18	19	0.02728	0.0176657
--	--	--	--	--	19	18	19	0.02728	0.0176657
--	--	--	--	--	19	19	20	-0.0676561	-0.0161366
--	--	--	--	--	20	19	20	-0.0676561	-0.0161366
--	--	--	--	--	20	10	20	0.0886135	0.0252389
--	--	--	--	--	21	10	21	0.157434	0.0931937
--	--	--	--	--	21	21	22	-0.0202004	-0.0241708
--	--	--	--	--	22	10	22	0.0756292	0.0409909
--	--	--	--	--	22	21	22	-0.0202004	-0.0241708
--	--	--	--	--	22	22	24	0.0565438	0.0189744
--	--	--	--	--	23	15	23	0.0510092	0.0383417
--	--	--	--	--	23	23	24	0.0185591	0.021467
--	--	--	--	--	24	22	24	0.0565438	0.0189744
--	--	--	--	--	24	23	24	0.0185591	0.021467
--	--	--	--	--	24	24	25	-0.0125724	-0.0276312
--	--	--	--	--	25	24	25	-0.0125724	-0.0276312
--	--	--	--	--	25	25	26	0.0354161	0.0236078
--	--	--	--	--	25	25	27	-0.0481832	-0.0515615
--	--	--	--	--	26	25	26	0.0354161	0.0236078
--	--	--	--	--	27	25	27	-0.0481832	-0.0515615
--	--	--	--	--	27	28	27	0.174	-0.0169484
--	--	--	--	--	27	27	29	0.0619027	0.0166509
--	--	--	--	--	27	27	30	0.070957	0.0166505
--	--	--	--	--	28	28	27	0.174	-0.0169484
--	--	--	--	--	28	8	28	0.043405	0.00177545
--	--	--	--	--	28	6	28	0.138712	0.0431573
--	--	--	--	--	29	27	29	0.0619027	0.0166509
--	--	--	--	--	29	29	30	0.0370854	0.0061221
--	--	--	--	--	30	27	30	0.070957	0.0166505
--	--	--	--	--	30	29	30	0.0370854	0.0061221

Note: - The line measurements and vi, ∂i are duplicated at the node-area but for ISE it is not necessary.

TABLE 2.1.5: TRANSFORMER DATA FOR 30 BUS SYSTEM

Transformer Number	Between Buses	Tap Setting
1	6 - 9	1.0155
2	6 – 10	0.9629
3	4 -12	1.0129
4	28 - 27	0.9581

TABLE 2.1.6: SHUNT CAPACITOR DATA FOR 30 BUS SYSTEM

Bus Number	Susceptance
10	0.19
24	0.04

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