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On Some Mixed Polynomial Exponential Diophantine Equation: $\alpha^n+\beta^n+a(\alpha^s\pm\beta^s)^m+D=r(u^k+v^k+w^k)$ with α and β Consecutive

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Let $a, \alpha, \beta, r, u, v, w$ and D be any integers and suppose that n, m, s and k are non-negative exponent. In this paper, the diophantine equation $\alpha^n+\beta^n+a(\alpha^s\pm\beta^s)^m+D=r(u^k+v^k+w^k)$ is developed and investigated for integer solution and its various polynomial identities. Moreover, the study formulates some conjectures for the title equation.

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1 Introduction

The study of Mixed polynomial exponential diophantine equation and integer decomposition into sums of powers are classical and has been a subject of considerable attention in recent past. Perhaps, may be because of the fact that, the study of integer decomposition has a direct application in the field of cryptography. Most researchers seems to have devoted their attention on Ramanujan Nagell Equation $x^2 + D = AB^n$ where x, n, A and B are variables and D is a fixed integer and sums of powers. For recent work on polynomial equations of sums of powers the reader may survey [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] and for detailed recap on Ramanujan Nagell Equation the reader may refer to [14, 15, 16, 17, 18, 19, 20, 21]. In most of this studies, the literature involving mixed polynomial and sums of powers is still hardily available. Moreover, documented results on diophantine equation $x^n + y^n + a(x^s \pm y^s)^m + D = r(u^k + v^k + w^k)$ proposed in this research is not known. This study is therefore, set to introduce and develop the formula $x^n + y^n + a(x^s \pm y^s)^m + D = r(u^k + v^k + w^k)$.

2 Main Results

The following assumptions will apply in this research. All numbers will be treated as integers, and it will be assumed that β is greater than α.

Conjecture 2.1. *For any integer $\beta > \alpha$ and exponent $m, n, k \geq 2$ and $s \geq 1$, there exist integers a, u, v, w and r such that*

$$\alpha^n + \beta^n + a(\alpha^s \pm \beta^s)^m + D = r(u^k + v^k + w^k) \cdots (1)$$

where D is an integer

In the sequel, we begin by constructing some solution of conjecture 2.1. We prioritize, determination of the unknowns a, m, n, r, s, k and D for which the equation $\alpha^n + \beta^n + a(\alpha^s \pm \beta^s)^m + D = r(u^k + v^k + w^k)$ has solution. The following cases has been considered. That is, $(a, m, n, k, r, s, D) = (1, 4, 4, 2, 2, 1, 0), (a, m, n, k, r, s, D) = (1, 4, 4, 2, 2, 1, 2)$.

Theorem 2.2. *Consider equation (1) satisfying the condition $(a, m, n, k, r, s, D) = (1, 4, 4, 2, 2, 1, 0)$. Then the diophantine equation $\alpha^4 + \beta^4 + (\alpha + \beta)^4 = 2(u_1^2 + v_1^2 + w_1^2)$ has solution in integers if α and β are consecutive*

Proof. Suppose α and β are consecutive integers and Consider the equation $\alpha^4 + \beta^4 + (\alpha+\beta)^4 = 2(u_1^2 + v_1^2 + w_1^2)$. The L.H.S expressed as $\alpha^4 + \beta^4 + (\alpha+\beta)^4 = \alpha^4 + (\alpha+1)^4 + (2\alpha+1)^4$ simplifies to $18\alpha^4 + 36\alpha^3 + 30\alpha^2 + 12\alpha + 2$. Rewriting the equation $18\alpha^4 + 36\alpha^3 + 30\alpha^2 + 12\alpha + 2 = 2(u_1^2 + v_1^2 + w_1^2)$ and dividing both sides by 2 we get $9\alpha^4 + 18\alpha^3 + 15\alpha^2 + 6\alpha + 1 = u_1^2 + v_1^2 + w_1^2$. To determine the value of u_1, v_1 and w_1 assume $u_1 = a\alpha^2 + b\alpha + c, v_1 = d\alpha^2 + e\alpha + f$ and $w_1 = g\alpha^2 + h\alpha + i$. Thus, $u_1^2 + v_1^2 + w_1^2 = (a\alpha^2 + b\alpha + c)^2 + (d\alpha^2 + e\alpha + f)^2 + (g\alpha^2 + h\alpha + i)^2 = a^2\alpha^4 + 2ab\alpha^3 + (2ac + b^2)\alpha^2 + 2bc\alpha + c^2 + d^2\alpha^4 + 2de\alpha^3 + (2df + e^2)\alpha^2 + 2ef\alpha + f^2 + g^2\alpha^4 + 2gh\alpha^3 + (2gi + h^2)\alpha^2 + 2hi\alpha + i^2 = (a^2 + d^2 + g^2)\alpha^4 + (2ab + 2de + 2gh)\alpha^3 + (2ac + b^2 + 2df + e^2 + 2gi + h^2)\alpha^2 + (2bc + 2ef + 2hi)\alpha + (c^2 + f^2 + i^2) = 9\alpha^4 + 18\alpha^3 + 15\alpha^2 + 6\alpha + 1$. Matching the coefficient we have

$$\begin{cases} a^2 + d^2 + g^2 = 9 \cdots (i), \\ 2ab + 2de + 2gh = 18 \cdots (ii), \\ 2ac + b^2 + 2df + e^2 + 2gi + h^2 = 15 \cdots (iii), \\ 2bc + 2ef + 2hi = 6 \cdots (iv), \\ c^2 + f^2 + i^2 = 1 \cdots (v). \end{cases}$$

Clearly, the system has 9 variables and 5 equation, thus there is no viable method to solve the system. Hence, we result to method of inspection. To solve the system we find the possible integer values $(a, b, c, d, e, f, g, h, i)$ that satisfy the system. We shall use step by step approach to determine correctly the solution set. From equation (1), $a^2 + d^2 + g^2 = 9$. Assume $a = 1$, thus $d^2 + g^2 = 8$. We need to find integer values for which $d^2 + g^2 = 8$. The only positive integer values are $d = 2$ and $g = 2$. Hence, $a^2 + d^2 + g^2 = 1^2 + 2^2 + 2^2 = 9$. Substituting the solution set $(a, d, g) = (1, 2, 2)$ into equation (ii) we obtain $2b + 4e + 4h = 18$. Dividing both sides by 2 we obtain $b + 2e + 2h = 9$. Need to find the solution set (b, e, h) which satisfy $b + 2e + 2h = 9$. Letting $b = 1, e = 1, h = 3$ we have $1 + 2(1) + 2(3) = 9$. Thus, $(b, e, h) = (1, 1, 3)$ is a solution. Substituting the solution $(a, d, g, b, e, h) = (1, 2, 2, 1, 1, 3)$ in equation (iii) we have $2c + 4f + 4i = 4$. Assuming $c = 0, f = 0, i = 1$ we get $2(0) + 4(0) + 4(1) = 4$. Thus $(c, f, i) = (0, 0, 1)$ is a solution. Since all the solution set have been determined i.e $(a, d, g, b, e, h, c, f, i) = (1, 2, 2, 1, 1, 3, 0, 0, 1)$ we consider this solution into equation (iv) and (v). Considering equation (iv),$2bc + 2ef + 2hi = 2(1)(0) + 2(1)(0) + 2(3)(1) = 6$. Hence, equation (iv) is satisfied. Finally, considering equation (v) we have $c^2 + f^2 + i^2 = 0^2 + 0^2 + 1^2 = 1$ which satisfies the equation. Consequently, $u_1 = \alpha^2 + \alpha, v_1 = 2\alpha^2 + \alpha$ and $w_1 = 2\alpha^2 + 3\alpha + 1$. Since u_1, v_1 and w_1 are known the result easily follows. $\square$

2.1 Examples

In this subsection, we provide some examples to argument our results in Theorem 2.1 for case (i).

Table 1. $\alpha^4 + \beta^4 + (\alpha + \beta)^4 = I = 2(u^2 + v^2 + w^2)$

α^4	β^4	$(\alpha+\beta)^4$	I	$u_1^2 = (\alpha^2 + \alpha)^2$	$v_1^2 = (2\alpha^2 + \alpha)^2$	$w_1^2 = (2\alpha^2 + 3\alpha + 1)^2$	
1	16	81	98	4	9	36	
16	81	625	722	36	100	225	
81	256	2401	2738	144	441	784	
256	625	6561	7442	400	1296	2025	
625	1296	14641	16562	900	3025	4356	
1296	2401	28561	32258	1764	6084	8281	
2401	4096	50625	57122	3136	11025	14400	

Theorem 2.3. *Consider equation* (1) *satisfying the condition* $(a, m, n, k, r, s, D) = (1, 4, 4, 2, 2, 1, 2)$. *Then the diophantine equation* $\alpha^4 + \beta^4 + (\alpha + \beta)^4 + 2 = 2(u^2 + v^2 + w^2)$ *has solution in integers if* α *and* β *are consecutive*

Proof. Suppose α and β are consecutive integers and Consider the equation $\alpha^4+\beta^4+(\alpha+\beta)^4+2 = 2(u^2+v^2+w^2)$. The L.H.S expressed as $\alpha^4+\beta^4+(\alpha+\beta)^4+2 = \alpha^4+(\alpha+1)^4+(2\alpha+1)^4+2$ simplifies to $18\alpha^4+36\alpha^3+30\alpha^2+12\alpha+4$. Rewriting the equation $18\alpha^4 + 36\alpha^3 + 30\alpha^2 + 12\alpha + 4 = 2(u^2 + v^2 + w^2)$ and dividing both sides by 2 we get $9\alpha^4 + 18\alpha^3 + 15\alpha^2 + 6\alpha + 2 = u^2 + v^2 + w^2$. To determine the value of u, v and w assume $u = a\alpha^2 + b\alpha + c, v = d\alpha^2+e\alpha+f$ and $w = g\alpha^2+h\alpha+i$. Thus, $u^2+v^2+w^2 = (a\alpha^2+b\alpha+c)^2+(d\alpha^2+e\alpha+f)^2+(g\alpha^2+h\alpha+i)^2 = a^2\alpha^4+2ab\alpha^3+(2ac+b^2)\alpha^2+2bc\alpha+c^2+d^2\alpha^4+2de\alpha^3+(2df+e^2)\alpha^2+2ef\alpha+f^2+g^2\alpha^4+2gh\alpha^3+(2gi+h^2)\alpha^2+2hi\alpha+i^2 = (a^2+d^2+g^2)\alpha^4+(2ab+2de+2gh)\alpha^3+(2ac+b^2+2df+e^2+2gi+h^2)\alpha^2+(2bc+2ef+2hi)\alpha+(c^2+f^2+i^2) = 9\alpha^4 + 18\alpha^3 + 15\alpha^2 + 6\alpha + 1$. Matching the coefficient we have

$$\begin{cases} a^2 + d^2 + g^2 = 9 \cdots (i), \\ 2ab + 2de + 2gh = 18 \cdots (ii), \\ 2ac + b^2 + 2df + e^2 + 2gi + h^2 = 15 \cdots (iii), \\ 2bc + 2ef + 2hi = 6 \cdots (iv), \\ c^2 + f^2 + i^2 = 2 \cdots (v). \end{cases}$$

Clearly, the system has 9 variables and 5 equation, thus there is no viable method to solve the system. Hence, we result to method of inspection. To solve the system we find the possible integer values $(a, b, c, d, e, f, g, h, i)$ that satisfy the system. We shall use step by step approach to determine correctly the solution set. From equation (1), $a^2 + d^2 + g^2 = 9$. Assume $a = 1$, thus $d^2 + g^2 = 8$. We need to find integer values for which $d^2 + g^2 = 8$. The only positive integer values are $d = 2$ and $g = 2$. Hence, $a^2 + d^2 + g^2 = 1^2 + 2^2 + 2^2 = 9$. Substituting the solution set $(a, d, g) = (1, 2, 2)$ into equation (ii) we obtain $2b + 4e + 4h = 18$. Dividing both sides by 2 we obtain $b + 2e + 2h = 9$. Need to find the solution set (b, e, h) which satisfy $b + 2e + 2h = 9$. Letting $b = 1, e = 2, h = 2$ we have $1 + 2(2) + 2(2) = 9$. Thus, $(b, e, h) = (1, 2, 2)$ is a solution. Substituting the solution $(a, d, g, b, e, h) = (1, 2, 2, 1, 2, 2)$ in equation (iii) we have $2c + 4f + 4i = 6$. Assuming $c = 1, f = 0, i = 1$ we get $2(1) + 4(0) + 4(1) = 6$. Thus $(c, f, i) = (1, 0, 1)$ is a solution. Since all the solution set have been determined i.e $(a, d, g, b, e, h, c, f, i) = (1, 2, 2, 1, 1, 2, 1, 0, 1)$ we consider this solution into equation (iv) and (v). Considering equation (iv),$2bc + 2ef + 2hi = 2(1)(1) + 2(1)(0) + 2(2)(1) = 6$. Hence, equation (iv) is satisfied. Finally, considering equation (v) we have $c^2 + f^2 + i^2 = 1^2 + 0^2 + 1^2 = 2$ which satisfies the equation. Consequently, $u = \alpha^2 + \alpha + 1, v = 2\alpha^2 + 2\alpha$ and $v = 2\alpha^2 + 2\alpha + 1$. Since u, v and w are known the result easily follows. ☐

In this subsection, we provide some examples to argument our results in Theorem 2.3 for case (ii).

Table 2. $\alpha^4 + \beta^4 + (\alpha + \beta)^4 + 2 = I_1 = 2(u^2 + v^2 + w^2)$

α^4	β^4	$(\alpha+\beta)^4 + 2$	I_1	$u^2 = (\alpha^2 + \alpha + 1)^2$	$v^2 = (\alpha^2 + 2\alpha)^2$	$w^2 = (2\alpha^2 + 2\alpha + 1)^2$	
1	16	83	100	9	16	25	
16	81	627	724	49	144	169	
81	256	2403	2740	169	576	625	
256	625	6563	7444	441	1600	1681	
625	1296	14643	16564	961	3600	7225	
1296	2401	28563	32260	1849	7056	12769	
2401	4096	50627	57124	3249	12544	21025	

Theorem 2.4. *Consider equation* (1) *satisfying the condition* $(a, m, n, k, r, s, D) = (1, 4, 4, 2, 2, 1, 0)$. *Then the diophantine equation* $\alpha^4 + \beta^4 + (\beta - \alpha)^4 = 2(u_2^2 + v_2^2 + w_2^2)$ *has solution in integers if* α *and* β *are consecutive*

Proof. Suppose α and β are consecutive integers and Consider the equation $\alpha^4 + \beta^4 + (\beta - \alpha)^4 = 2(u_2^2 + v_2^2 + w_2^2)$. The L.H.S expressed as $\alpha^4 + \beta^4 + (\beta - \alpha)^4 = \alpha^4 + (\alpha + 1)^4 + 1^4$ simplifies to $2\alpha^4 + 4\alpha^3 + 6\alpha^2 + 4\alpha + 2$. Rewriting the equation $2\alpha^4 + 4\alpha^3 + 6\alpha^2 + 4\alpha + 2 = 2(u_2^2 + v_2^2 + w_2^2)$ and dividing both sides by 2 we get $\alpha^4 + 2\alpha^3 + 3\alpha^2 + 2\alpha + 1 = u_2^2 + v_2^2 + w_2^2$. To determine the value of u_2, v_2 and w_2 assume $u_2 = a\alpha^2 + b\alpha + c, v_2 = d\alpha^2 + e\alpha + f$ and $w_2 = g\alpha^2 + h\alpha + i$. Thus, $u_2^2 + v_2^2 + w_2^2 = (a\alpha^2 + b\alpha + c)^2 + (d\alpha^2 + e\alpha + f)^2 + (g\alpha^2 + h\alpha + i)^2 = a^2\alpha^4 + 2ab\alpha^3 + (2ac + b^2)\alpha^2 + 2bc\alpha + c^2 + d^2\alpha^4 + 2de\alpha^3 + (2df + e^2)\alpha^2 + 2ef\alpha + f^2 + g^2\alpha^4 + 2gh\alpha^3 + (2gi + h^2)\alpha^2 + 2hi\alpha + i^2 = (a^2 + d^2 + g^2)\alpha^4 + (2ab + 2de + 2gh)\alpha^3 + (2ac + b^2 + 2df + e^2 + 2gi + h^2)\alpha^2 + (2bc + 2ef + 2hi)\alpha + (c^2 + f^2 + i^2) = \alpha^4 + 2\alpha^3 + 3\alpha^2 + 2\alpha + 1$. Matching the coefficient we have

$$\begin{cases} a^2 + d^2 + g^2 = 1 \cdots (i), \\ 2ab + 2de + 2gh = 2 \cdots (ii), \\ 2ac + b^2 + 2df + e^2 + 2gi + h^2 = 3 \cdots (iii), \\ 2bc + 2ef + 2hi = 2 \cdots (iv), \\ c^2 + f^2 + i^2 = 1 \cdots (v). \end{cases}$$

Clearly, the system has 9 variables and 5 equation, thus there is no viable method to solve the system. Hence, we result to method of inspection. To solve the system we find the possible integer values $(a, b, c, d, e, f, g, h, i)$ that satisfy the system. We shall use step by step approach to determine correctly the solution set. From equation (1), $a^2 + d^2 + g^2 = 1$. Assume $a = 0$, thus $d^2 + g^2 = 1$. We need to find integer values for which $d^2 + g^2 = 1$. The integer values are $d = 0$ and $g = 1$. Hence, $a^2 + d^2 + g^2 = 0^2 + 0^2 + 1^2 = 1$. Substituting the solution set $(a, d, g) = (0, 0, 1)$ into equation (ii) we obtain $2gh = 2$. Dividing both sides by 2 we obtain $gh = 1$. Clearly, $g = 1$ and $h = 1$. Thus, $(g, h) = (1, 1)$ is a solution. Substituting the solution $(a, d, g, b, h) = (0, 0, 1, 1, 1)$ in equation (iii) we have $b^2 + e^2 = 2$. Assuming $b = 1, e = 1$ we get $1^2 + 1^2 = 2$. Thus $(b, e) = (1, 1)$ is a solution. Substituting the solution set $(a, d, g, b, e, h) = (0, 0, 1, 1, 1, 1)$ into equation (iv) we have $c = 0$. Finally, considering equation (v) we have $c^2 + f^2 + i^2 = 0^2 + 1^2 + 0^2 = 1$ which satisfies the equation. Consequently, $u_2 = \alpha, v_2 = \alpha + 1$ and $w_2 = \alpha^2 + \alpha$. Since u_2, v_2 and w_2 are known the result easily follows. □

In this subsection, we provide some examples to argument our results in Theorem 2.4 for case (i).

Table 3. $\alpha^4 + \beta^4 + (\beta - \alpha)^4 = I_2 = 2(u_2^2 + v_2^2 + w_2^2)$

α^4	β^4	$(\beta-\alpha)^4$	I_2	$u_2^2 = \alpha^2$	$v_2^2 = (\alpha+1)^2$	$w_2^2 = (\alpha^2+\alpha)^2$	
1	16	1	18	1	4	4	
16	81	1	98	4	9	36	
81	256	1	338	9	16	144	
256	625	1	882	16	25	400	
625	1296	1	1922	25	36	900	
1296	2401	1	3698	36	49	1764	
2401	4096	1	6498	49	64	3136	

3 Conclusion

To sum up, this research has provided integral solution for the diophantine equation $\alpha^n + \beta^n + a(\alpha^s \pm \beta^s)^m + D = r(u^k + v^k + w^k)$ where α and β are consecutive integers. Future research may investigate the same families of the diophantine equation with different exponent and the difference between alpha and beta greater or equal to 2.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

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Competing Interests

Author has declared that no competing interests exist.

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