Modulational Instability of Ion Acoustic Waves in a Magnetized Plasma Consisting of Isothermally Distributed Hot and Cold Electrons

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

Using the standard Reductive Perturbation Method a nonlinear Schrödinger equation is derived to study the modulational instability of small amplitude ion acoustic waves in a collisionless magnetized plasma composed of adiabatic warm ions, Maxwell-Boltzmann distribution of hot electrons as well as Maxwell-Boltzmann distribution of cold electrons, and the plasma system immersed in an external uniform static magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) propagating along the $z-$axis. The instability condition and the maximum growth rate of instability have been investigated analytically as well as numerically. We have studied the effect of each parameter of the present plasma system on the maximum growth rate of instability. In particular, it is found that the maximum growth rate of instability decreases with the increasing value of the ion cyclotron frequency with some set of values of the parameters associated with the present plasma system. Again, we have seen that the instability region decreases with the increasing value of the ion cyclotron frequency.

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1 INTRODUCTION

Numerous satellite observations in the Earth's magnetosphere, viz., FAST at the auroral region [1, 2, 3, 4, 5], Viking Satellite [6, 7], S3-3 Satellite [8], GEOTAIL [9] and POLAR mission [10, 11, 5] confirm the coexistence of cold and hot electron plasmas. Studies of two - electron - temperature plasmas by these satellite observations of moving localized potential variation regions are very interesting field of research. On the basis of laboratory experiments [12, 13, 14, 15], we can say that two different temperatures (e.g., hot and cold) of electrons in plasma are also a very interesting field of research. Several authors [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] investigated the nonlinear properties of arbitrary amplitude ion acoustic (IA) solitary waves in an unmagnetized/magnetized plasma consisting of Maxwell-Boltzmann distribution of hot electrons and Maxwell-Boltzmann distribution of cold electrons.

Kourakis and Shukla [29] investigated the oblique modulational instability (MI) of IA envelop solitary waves in a collisionless unmagnetized plasma consisting of cold ions, isothermally distributed cold and hot electrons. Esfandyari and Asgari [30] investigated the oblique MI of IA waves in a collisionless unmagnetized plasma system consisting of warm ions, isothermally distributed cold and hot electrons. Esfandyari et al. [31] investigated the MI of IA waves in a collisionless unmagnetized plasma system consisting of adiabatic warm ions, two distinct temperatures of isothermal electrons. Dalui et al. [32] investigated the MI of IA waves in a collisionless unmagnetized plasma system consisting of warm adiabatic ions, nonthermal distribution of hot electrons and Maxwell-Boltzmann distribution of cold electrons.

In this paper, we have studied the modulational instability of IA waves in a collisionless magnetized plasma composed of adiabatic warm ions, Maxwell-Boltzmann distribution of hot electrons as well as Maxwell-Boltzmann

distribution of cold electrons. Here, also we have considered the external uniform static magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) propagating along the $z-$axis. Using the standard Reductive Perturbation Method (RPM) [33, 34], we have derived a three dimensional nonlinear Schrödinger equation (NLSE).

2 BASIC EQUATIONS

We consider a nonlinear behaviour of ion acoustic waves in a collisionless magnetized plasma composed of adiabatic warm ions, Maxwell-Boltzmann distribution of hot electrons as well as Maxwell-Boltzmann distribution of cold electrons, and the plasma system immersed in an external uniform static magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) propagating along the $z-$axis. Therefore, the basic equations of the present plasma system are given by

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \qquad (2.1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi + \omega_c(\mathbf{u} \times \hat{\mathbf{z}}) - \frac{\sigma}{n}\nabla p, \quad (2.2)$$

$$\nabla^2\phi = n_{ce} + n_{se} - n, \qquad (2.3)$$

$$p = n^\gamma, \qquad (2.4)$$

where $\nabla \equiv (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.

Here n is number density of ions and normalized by n_0 (unperturbed ion number density), n_{ce} is number density of hot electrons and normalized by n_0, n_{se} is number density of cold electrons and normalized by n_0, ω_c is ion cyclotron frequency and normalized by $\omega_{pi}(= \sqrt{4\pi n_0 e^2/m})$, $\mathbf{u} = (u, v, w)$ is ion fluid velocity and normalized by c_s $(= \sqrt{K_B T_{ef}/m})$, p is ion pressure and normalized by $n_0 K_B T_i$, ϕ is electrostatic potential and normalized by $K_B T_{ef}/e$, (x, y, z) is spatial variables and normalized by λ_{Df} $(= \sqrt{K_B T_{ef}/4\pi n_0 e^2})$, and t is time and normalized by ω_{pi}^{-1}, where $\sigma = T_i/T_{ef}$ and $\gamma = \frac{5}{3}$. Again K_B, m, $-e$ and T_i are, respectively, the Boltzmann constant, mass of an ion, charge of an electron

and average ion temperature. And the expression of T_{ef} as follows:

$$T_{ef} = (n_{c0} + n_{s0}) \left[\frac{n_{c0}}{T_{ce}} + \frac{n_{s0}}{T_{se}} \right]^{-1}, \qquad (2.5)$$

where n_{c0}, n_{s0}, T_{ce} and T_{se} are the unperturbed number density of hot electrons, the unperturbed number density of cold electrons, average temperature of hot electrons and average temperature of cold electrons respectively.

After the normalization, the expressions of the number densities of isothermally distributed hot and cold electrons can be written by as follow:

$$n_{ce} = \bar{n}_{c0} \exp[\sigma_c \phi], \qquad (2.6)$$

$$n_{se} = \bar{n}_{s0} \exp[\sigma_s \phi], \qquad (2.7)$$

where $\bar{n}_{c0} = \frac{n_{c0}}{n_0}$, $\bar{n}_{s0} = \frac{n_{s0}}{n_0}$, $\sigma_c = \frac{T_{ef}}{T_{ce}}$ and $\sigma_s = \frac{T_{ef}}{T_{se}}$.

Here, we consider the charge neutrality condition of the plasma system as follows:

$$n_{c0} + n_{s0} = n_0. \qquad (2.8)$$

Using (2.8), the equation (2.5) can be expressed as

$$\bar{n}_{c0}\sigma_c + \bar{n}_{s0}\sigma_s = 1. \qquad (2.9)$$

Using the equations (2.5), (2.8) and (2.9), we get

$$\bar{n}_{s0} = \frac{n_{sc}}{1 + n_{sc}}, \quad \bar{n}_{c0} = \frac{1}{1 + n_{sc}}, \qquad (2.10)$$

$$\sigma_s = \frac{1 + n_{sc}}{\sigma_{sc} + n_{sc}}, \quad \sigma_c = \sigma_{sc}\frac{1 + n_{sc}}{\sigma_{sc} + n_{sc}}, \qquad (2.11)$$

where $\sigma_{sc} = \frac{T_{se}}{T_{ce}}$ and $n_{sc} = \frac{n_{s0}}{n_{c0}}$.

Expanding both n_{ce} and n_{se} as given in (2.6) and (2.7), using the equations (2.8) - (2.11) and keeping the terms up to ϕ^3, the Poisson equation (2.3) can be written as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \qquad (2.12)$$
$$= h_0 + h_1\phi + h_2\phi^2 + h_3\phi^3 - n,$$

where h_0, h_1, h_2 and h_3 are given by

$$h_0 = 1, h_1 = \left[\bar{n}_{s0}\sigma_s + \bar{n}_{c0}\sigma_c \right], \qquad (2.13)$$

$$h_2 = \frac{1}{2} \left[\bar{n}_{s0}\sigma_s^2 + \bar{n}_{c0}\sigma_c^2 \right], \qquad (2.14)$$

$$h_3 = \frac{1}{6} \left[\bar{n}_{s0}\sigma_s^3 + \bar{n}_{c0}\sigma_c^3 \right]. \qquad (2.15)$$

3 DERIVATION OF THE NLSE

To investigate the MI of the IA waves in a collisionless magnetized warm plasma composed of adiabatic warm ions, Maxwell-Boltzmann distribution of hot electrons as well as Maxwell-Boltzmann distribution of cold electrons, immersed in an external uniform static magnetic field propagating along the $z-$axis, we have considered the following stretching of the spatial and time variables as follow:

$$\xi = \epsilon x, \eta = \epsilon y, \zeta = \epsilon(z - V_g t), \tau = \epsilon^2 t, \qquad (3.1)$$

where V_g is a constant and ϵ is a small parameter. We consider the perturbation of the field quantities as follow:

$$f = f^{(0)} + \sum_{l=1}^{\infty} \epsilon^l \sum_{a=-\infty}^{\infty} f_a^{(l)}(\xi, \eta, \zeta, \tau) \exp[ia\psi], \qquad (3.2)$$

$$s = s^{(0)} + \sum_{l=1}^{\infty} \epsilon^{l+1} \sum_{a=-\infty}^{\infty} s_a^{(l)}(\xi, \eta, \zeta, \tau) \exp[ia\psi], \qquad (3.3)$$

where

$$\psi = kz - \omega t, \qquad (3.4)$$

k and ω are the wave number and the wave frequency of the IA waves respectively. Also, $f = n$, w, ϕ and $s = u$, v with $f^{(0)} = [1, 0, 0]^T$ and $s^{(0)} = [0, 0]^T$. Here also we have considered the notations: $f_{-a}^{(l)} = \bar{f}_a^{(l)}$ and $s_{-a}^{(l)} = \bar{s}_a^{(l)}$ where 'bar' represents the complex conjugate. Again, for $a = 0$ and $-a = 0$ we obtain only one term in both the perturbation expansions (3.2) and (3.3), and therefore, we can consider $\bar{f}_0^{(l)} = \bar{s}_0^{(l)} = 0$ for all l.

We have used the consistency conditions $(n_0^{(1)}, u_0^{(1)}, v_0^{(1)}, w_0^{(1)}, \phi_0^{(1)}) = (0, 0, 0, 0, 0)$ and $(n_a^{(l)}, u_a^{(l)}, v_a^{(l)}, w_a^{(l)}, \phi_a^{(l)}) = (0, 0, 0, 0, 0)$ for $l < |a|$ to make a one - one correspondence between the RPM and the multiple scale perturbation method [35, 36].

Substituting the perturbation expansions for n, u, v, w and ϕ, into the equations (2.1), (2.2), (2.4) and (2.12) and collecting the terms of different powers of ϵ, we get a sequence of equations of different orders. From each equation of a particular order, one can generate another sequence of equations for different harmonics by changing the values of a.

3.1 First Order :

Collecting the first order ($O(\epsilon) = 1$) zeroth harmonic ($a = 0$) equations of continuity equation for ions, the z-component of motion equation for ions and the Poisson equation, we can conclude that the first order zeroth harmonic equations are trivially satisfied according to the first consistency condition.

Solving the first order ($O(\epsilon) = 1$) first harmonic ($a = 1$) equations collecting from the continuity equation for ions and the z-component of motion equation for ions, we get

$$n_1^{(1)} = \frac{k^2}{W^2}\phi_1^{(1)}, w_1^{(1)} = \frac{k\omega}{W^2}\phi_1^{(1)}, \qquad (3.5)$$

where $W^2 = \omega^2 - \sigma\gamma k^2$.

The first order ($O(\epsilon) = 1$) first harmonic ($a = 1$) equation collecting from the Poisson equation, we get

$$n_1^{(1)} = (k^2 + h_1)\phi_1^{(1)}. \qquad (3.6)$$

Solving the equation (3.6) and the first equation of (3.5), we get the following linear dispersion relation (LDR) of IA waves:

$$\frac{\omega^2}{k^2} = \frac{1}{k^2 + h_1} + \gamma\sigma. \qquad (3.7)$$

3.2 Second Order :

3.2.1 First harmonic

Solving the second order ($O(\epsilon) = 2$) first harmonic ($a = 1$) equations collecting from the continuity equation for ions and the z-component of motion equation for ions, we get

$$n_1^{(2)} = \frac{k^2}{W^2}\phi_1^{(2)} + \frac{2ik\omega(V_g k - \omega)}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\zeta}, \qquad (3.8)$$

$$w_1^{(2)} = \frac{k\omega}{W^2}\phi_1^{(2)} + \frac{i(V_g k - \omega)(\omega^2 + \sigma\gamma k^2)}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\zeta}. \qquad (3.9)$$

Again, solving the first harmonic ($a = 1$) equations collecting from the x-component and y-component of the motion equation for ions, we get

$$u_1^{(1)} = \frac{\omega^2}{W^2(\omega_c^2 - \omega^2)}\left[i\omega\frac{\partial\phi_1^{(1)}}{\partial\xi} - \omega_c\frac{\partial\phi_1^{(1)}}{\partial\eta}\right]. \quad (3.10)$$

$$v_1^{(1)} = \frac{\omega^2}{W^2(\omega_c^2 - \omega^2)}\left[i\omega\frac{\partial\phi_1^{(1)}}{\partial\eta} + \omega_c\frac{\partial\phi_1^{(1)}}{\partial\xi}\right]. \quad (3.11)$$

From the second order ($O(\epsilon) = 2$) first harmonic ($a = 1$) equation collecting from the Poisson equation, we get

$$n_1^{(2)} = (k^2 + h_1)\phi_1^{(2)} - 2ik\frac{\partial\phi_1^{(1)}}{\partial\zeta}. \qquad (3.12)$$

Substituting (3.8) in the equation (3.12) and eliminating $n_1^{(2)}$, we get

$$-\frac{k^2(k^2+h_1)}{W^2}\left\{\frac{\omega^2}{k^2} - \left(\gamma\sigma + \frac{1}{k^2+h_1}\right)\right\}\phi_1^{(2)} \qquad (3.13)$$
$$+\frac{2i\omega k^2}{W^4}\left(V_g - \frac{\omega^2 - W^4}{\omega k}\right)\frac{\partial\phi_1^{(1)}}{\partial\zeta} = 0.$$

The first term of the equation (3.13) is equal to zero according to LDR (3.7) and the second term of the equation (3.13) can be made equal to zero if V_g follows the following relation

$$V_g = \frac{\omega^2 - W^4}{\omega k} = \frac{k}{\omega}\left[\frac{h_1}{(k^2+h_1)^2} + \sigma\gamma\right]. \quad (3.14)$$

Now, differentiating the LDR (3.7) with respect to k, we get

$$\frac{\partial\omega}{\partial k} = \frac{k}{\omega}\left[\frac{h_1}{(k^2+h_1)^2} + \sigma\gamma\right]. \qquad (3.15)$$

We get from the equations (3.14) and (3.15)

$$V_g = \frac{\partial\omega}{\partial k}, \qquad (3.16)$$

and consequently, the equation (3.13) is identically satisfied if V_g is the group velocity of the IA waves.

3.2.2 Second harmonic

Solving the second order ($O(\epsilon) = 2$) second harmonic ($a = 2$) equations collecting from the continuity equation for ions, the z-component of the motion equation for ions and the Poisson equation, we get

$$(\phi_2^{(2)}, n_2^{(2)}, w_2^{(2)}) = (A_\phi, A_n, A_w)[\phi_1^{(1)}]^2, \quad (3.17)$$

where

$$A_\phi = -\frac{h_2}{3k^2} + \frac{k^2\omega^2}{2W^6} + g_1\sigma\gamma\frac{k^4}{6W^6}, \quad (3.18)$$

$$A_n = (4k^2 + h_1)A_\phi + h_2, \qquad (3.19)$$

$$A_w = \frac{\omega}{k}\left[A_n - \frac{k^4}{W^4}\right], \qquad (3.20)$$

and $g_1 = (\gamma - 2)$.

3.2.3 Zeroth harmonic

Solving the zeroth harmonic ($a = 0$) equations collecting from the continuity equation for ions, the z-component of the motion equation for ions and the Poisson equation, we get

$$(\phi_0^{(2)}, n_0^{(2)}, w_0^{(2)}) = (B_\phi, B_n, B_w)|\phi_1^{(1)}|^2, \quad (3.21)$$

where

$$B_\phi = \frac{g_1\gamma\sigma k^4 + k^2\omega(2kV_g + \omega) - 2h_2 W^4(V_g^2 - \gamma\sigma)}{W^4[h_1(V_g^2 - \sigma\gamma) - 1]}, \quad (3.22)$$

$$B_n = h_1 B_\phi + 2h_2, \quad (3.23)$$

$$B_w = V_g B_n - \frac{2\omega k^3}{W^4}. \quad (3.24)$$

3.3 Third Order : First Harmonic

Solving the third order ($O(\epsilon) = 3$) first harmonic ($a = 1$) equations collecting from the continuity equation for ions and the z-component of the motion equation for ions, we can express $n_1^{(3)}$ and $w_1^{(3)}$ as a function of $\phi_1^{(1)}$, $\phi_1^{(2)}$ and $\phi_1^{(3)}$ along with their different derivatives with respect to ξ, η, ζ and τ. In particular, $n_1^{(3)}$ can be written as follows:

$$n_1^{(3)} = \frac{k^2}{W^2}\phi_1^{(3)} + 2ik\frac{\omega(V_g k - \omega)}{W^4}\frac{\partial\phi_1^{(2)}}{\partial\zeta} - i\frac{2k^2\omega}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\tau}$$

$$+ \frac{\omega^4}{W^4(\omega_c^2 - \omega^2)}\Big(\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2} + \frac{\partial^2\phi_1^{(1)}}{\partial\eta^2}\Big)$$

$$- \frac{(V_g k - \omega)}{W^6}\Big(3V_g k\omega^2 - 3\sigma\gamma k^2\omega - \omega^3 + \sigma\gamma V_g k^3\Big)\frac{\partial^2\phi_1^{(1)}}{\partial\zeta^2}$$

$$+ \Big[2\frac{k^3\omega}{W^4}(A_w + B_w) + \frac{k^2}{W^4}(\omega^2 + \sigma\gamma g_1 k^2)(A_n + B_n) + \sigma\gamma g_2\frac{k^8}{W^8}\Big]|\phi_1^{(1)}|^2\phi_1^{(1)}, \quad (3.25)$$

where $g_2 = \frac{(\gamma-2)(\gamma-3)}{2}$.

The third order ($O(\epsilon) = 3$) first harmonic ($a = 1$) equation collecting from the Poisson equation, we get

$$n_1^{(3)} = (k^2 + h_1)\phi_1^{(3)} - 2ik\frac{\partial\phi_1^{(2)}}{\partial\zeta} - \frac{\partial^2\phi_1^{(1)}}{\partial\xi^2} - \frac{\partial^2\phi_1^{(1)}}{\partial\eta^2} - \frac{\partial^2\phi_1^{(1)}}{\partial\zeta^2}$$

$$+ \{2h_2(A_\phi + B_\phi) + 3h_3\}|\phi_1^{(1)}|^2\phi_1^{(1)}. \quad (3.26)$$

Now, eliminating $n_1^{(3)}$ from the equations (3.26) and (3.25), we get

$$- \frac{k^2(k^2 + h_1)}{W^2}\Big\{\frac{\omega^2}{k^2} - \Big(\gamma\sigma + \frac{1}{k^2 + h_1}\Big)\Big\}\phi_1^{(3)} + \frac{2i\omega k^2}{W^4}\Big(V_g - \frac{\omega^2 - W^4}{\omega k}\Big)\frac{\partial\phi_1^{(2)}}{\partial\zeta}$$

$$- i\frac{2k^2\omega}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\tau} + \Big(\frac{\omega^4}{W^4(\omega_c^2 - \omega^2)} + 1\Big)\Big(\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2} + \frac{\partial^2\phi_1^{(1)}}{\partial\eta^2}\Big)$$

$$+ \Big[1 - \frac{k^4(V_g - \frac{\omega}{k})}{W^6}(3V_g\frac{\omega^2}{k^2} - 3\gamma\sigma\frac{\omega}{k} - \frac{\omega^3}{k^3} + \gamma\sigma V_g)\Big]\frac{\partial^2\phi_1^{(1)}}{\partial\zeta^2}$$

$$+ \Big[2\frac{k^3\omega}{W^4}(A_w + B_w) + \frac{k^2}{W^4}(\omega^2 + \sigma\gamma g_1 k^2)(A_n + B_n) + \sigma\gamma g_2\frac{k^8}{W^8}$$

$$- 3h_3 - 2h_2(A_\phi + B_\phi)\Big]|\phi_1^{(1)}|^2\phi_1^{(1)} = 0. \quad (3.27)$$

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Using the equations (3.7) and (3.14), the equation (3.27) can be written as

$$i\frac{\partial \phi_1^{(1)}}{\partial \tau} + P\frac{\partial^2 \phi_1^{(1)}}{\partial \zeta^2} + Q|\phi_1^{(1)}|^2\phi_1^{(1)} - R\Big(\frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi_1^{(1)}}{\partial \eta^2}\Big) = 0, \tag{3.28}$$

where

$$P = -\frac{W^4}{2k^2\omega}\left[1 - \frac{k^4}{W^6}\Big(V_g - \frac{\omega}{k}\Big)\Big(3V_g\frac{\omega^2}{k^2} - 3\gamma\sigma\frac{\omega}{k} - \frac{\omega^3}{k^3} + \gamma\sigma V_g\Big)\right], \tag{3.29}$$

$$\begin{aligned} Q &= -\frac{W^4}{2k^2\omega}\Bigg[2\frac{k^3\omega}{W^4}(A_w + B_w) + \frac{k^2}{W^4}(\omega^2 + \sigma\gamma g_1 k^2)(A_n + B_n) + \sigma\gamma g_2\frac{k^8}{W^8} \\ &\quad - 3h_3 - 2h_2(A_\phi + B_\phi)\Bigg], \end{aligned} \tag{3.30}$$

$$R = \frac{W^4}{2k^2\omega}\left[\frac{\omega^4}{W^4(\omega_c^2 - \omega^2)} + 1\right]. \tag{3.31}$$

4 MODULATIONAL INSTABILITY

To study the modulational instability of IA waves in a collisionless magnetized plasma, we assume the IA wave is propagating along a direction having direction cosines (l_1, m_1, n_1) and consequently, we take the transformation

$$\xi' = l_1\xi + m_1\eta + n_1\zeta, \tau' = \tau, \tag{4.1}$$

where $l_1^2 + m_1^2 + n_1^2 = 1$.

Substituting (4.1) in equation (3.28) and dropping the prime from the independent variables ξ' and τ', we get

$$i\frac{\partial \phi_1^{(1)}}{\partial \tau} + P_1\frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + Q|\phi_1^{(1)}|^2\phi_1^{(1)} = 0, \tag{4.2}$$

where

$$\begin{aligned} P_1 &= [Pn_1^2 - R(l_1^2 + m_1^2)], \\ &= [(P + R)n_1^2 - R]. \end{aligned} \tag{4.3}$$

Now, we see that $\phi_1^{(1)} = \phi_0 e^{i\Delta\tau}$ is a steady state solution of the NLSE (4.2) if $\Delta = Q|\phi_0|^2$, where ϕ_0 is a constant.

Again, we take $\phi_1^{(1)}$ as

$$\phi_1^{(1)} = (\phi_0 + \delta\phi)e^{i\Delta\tau}, \text{ where } |\delta\phi| << |\phi_0|. \tag{4.4}$$

Substituting (4.4) into the equation (4.2) and finally, linearizing the equation with respect to the perturbed quantity $\delta\phi$, we get

$$i\frac{\partial \delta\phi}{\partial \tau} + P_1\frac{\partial^2 \delta\phi}{\partial \xi^2} + Q|\phi_0|^2(\delta\phi + \overline{\delta\phi}) = 0, \tag{4.5}$$

where complex conjugate of $\delta\phi$ is $\overline{\delta\phi}$.

Substituting $\delta\phi = U + iV$ into the equation (4.5) and then separating into real and imaginary parts, we get

$$-\frac{\partial V}{\partial \tau} + P_1\frac{\partial^2 U}{\partial \xi^2} + 2QU|\phi_0|^2 = 0, \tag{4.6}$$

$$\frac{\partial U}{\partial \tau} + P_1\frac{\partial^2 V}{\partial \xi^2} = 0, \tag{4.7}$$

where $U(\xi, \tau)$ and $V(\xi, \tau)$ are real functions of ξ and τ.

Again, we consider the form of $U(\xi, \tau)$ and $V(\xi, \tau)$ as given by the following equations:

$$U = U_0\exp\left[i(K\xi - \Omega\tau)\right] + c.c.\,, \tag{4.8}$$

$$V = V_0\exp\left[i(K\xi - \Omega\tau)\right] + c.c. \tag{4.9}$$

Substituting $U(\xi, \tau)$ and $V(\xi, \tau)$ into the equations (4.6) and (4.7), we get

$$i\Omega V_0 + (-P_1K^2 + 2Q|\phi_0|^2)U_0 = 0, \tag{4.10}$$

$$i\Omega U_0 + P_1K^2V_0 = 0. \tag{4.11}$$

For the non trivial solution of the above linear equations (4.10) and (4.11) for the unknown quantities U_0 and V_0, we get

$$\Omega^2 = [P_1K^2]^2\Big(1 - \frac{2Q|\phi_0|^2}{P_1K^2}\Big). \tag{4.12}$$

This is the modulated nonlinear dispersion relation.

If $1 - \frac{2Q|\phi_0|^2}{P_1 K^2} \geq 0$, then from the relation (4.12), we can get real values of Ω and consequently, the modulated IA wave is stable. From the expression of Ω^2 as given in the equation (4.12), we see that Ω^2 is strictly positive for $P_1 Q < 0$. Therefore, the modulated IA wave is always stable for all $P_1 Q < 0$. On the other hand, if $P_1 Q > 0$, then $\Omega^2 \geq 0$ when $K \geq K_c$ or $\Omega^2 < 0$ when $K < K_c$, where K_c is given by as follows:

$$K_c = \sqrt{\frac{2Q|\phi_0|^2}{P_1}}. \tag{4.13}$$

Therefore, we see that the modulated IA wave is stable, i.e., $\Omega^2 \geq 0$ when either $P_1 Q < 0$ or $K \geq K_c$ whenever $P_1 Q > 0$.

On the other hand, if $P_1 Q > 0$ and $K < K_c$ then $\Omega^2 < 0$ and all the roots of the equation (4.12) for the unknown Ω are purely imaginary and consequently, the modulated IA wave is unstable.

The modulational growth rate of instability (Γ) is given by

$$\Gamma = [P_1 K^2]^2 \left[\frac{2Q|\phi_0|^2}{P_1 K^2} - 1\right]^{1/2}. \tag{4.14}$$

Therefore, the maximum growth rate of instability (Γ_{max}) is obtained by $\frac{\partial \Gamma}{\partial K} = 0$. Thus, we have

$$\Gamma_{max} = |Q||\phi_0|^2, \tag{4.15}$$

when $K = \sqrt{\frac{Q|\phi_0|^2}{P_1}}$.
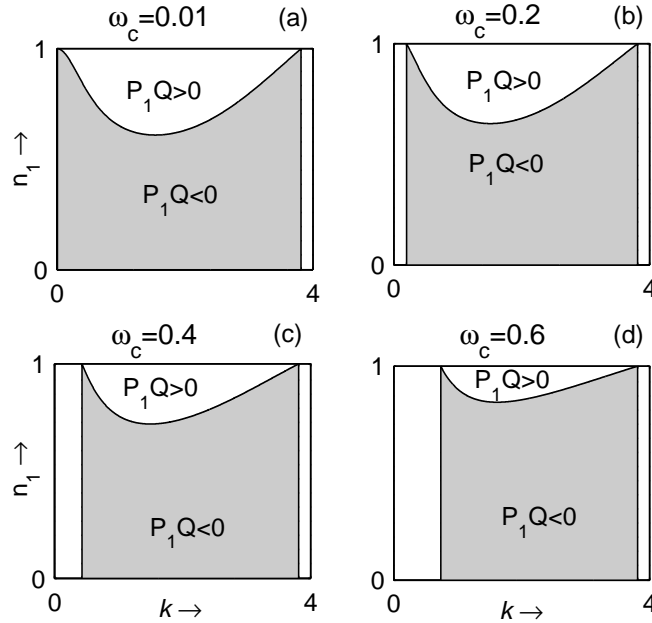
5 SUMMARY & DISCUSSIONS

Fig. 1. When $P_1 Q = 0$, n_1 has been plotted against k for different values of ω_c with $\gamma = 5/3$, $\sigma = 0.001$, $n_{sc} = 0.25$ and $\sigma_{sc} = 0.25$.
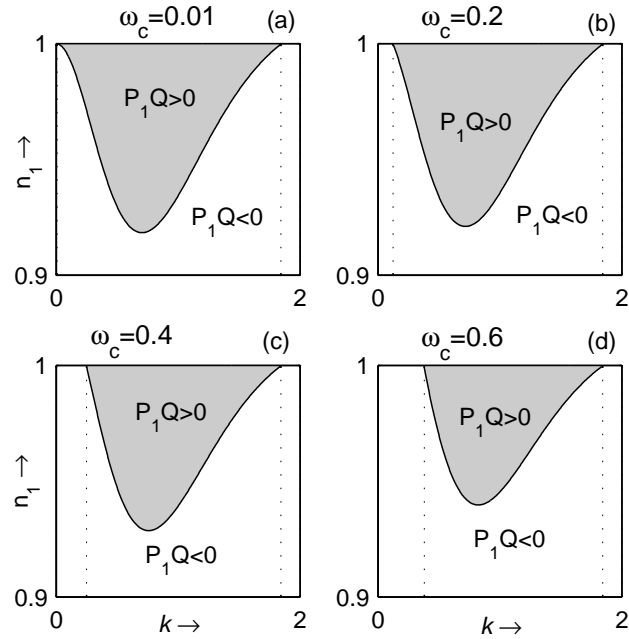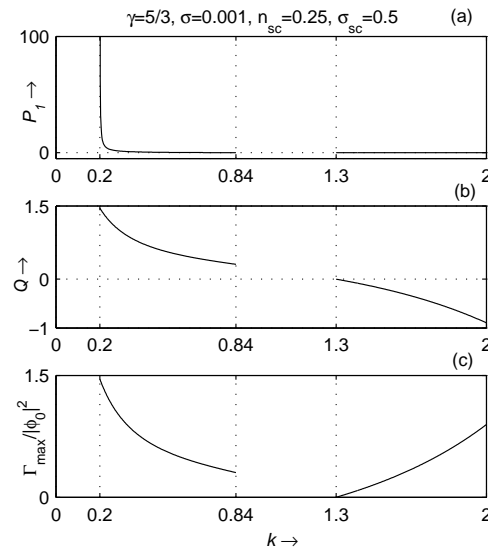
Fig. 2. When $P_1Q = 0$, n_1 **has been plotted against** k **for different values of** ω_c **with** $\gamma = 5/3$, $\sigma = 0.1$, $n_{sc} = 0.25$ **and** $\sigma_{sc} = 0.25$.

Fig. 3. P_1, Q **and** $\Gamma_{max}/|\phi_0|^2$ **have been plotted against** k **in (a), (b) and (c) respectively for** $n_1 = 0.7$ **and** $\omega_c = 0.2$.

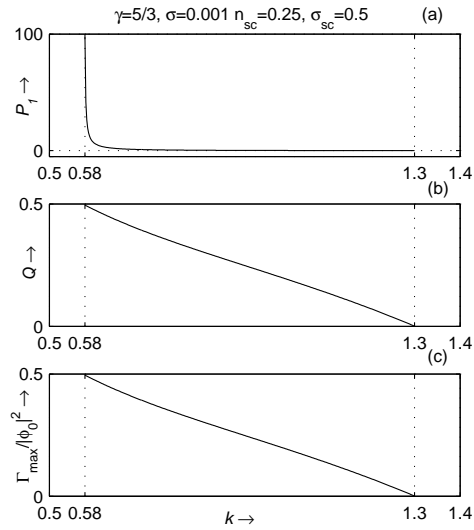**Fig. 4.** P_1**,** Q **and** $\Gamma_{max}/|\phi_0|^2$ **have been plotted against** k **in (a), (b) and (c) respectively for** $n_1 = 0.7$ **and** $\omega_c = 0.5$**.**

Fig. 5. P_1**,** Q **and** $\Gamma_{max}/|\phi_0|^2$ **have been plotted against** k **in (a), (b) and (c) respectively for** $n_1 = 0.25$ **and** $\omega_c = 0.2$**.**
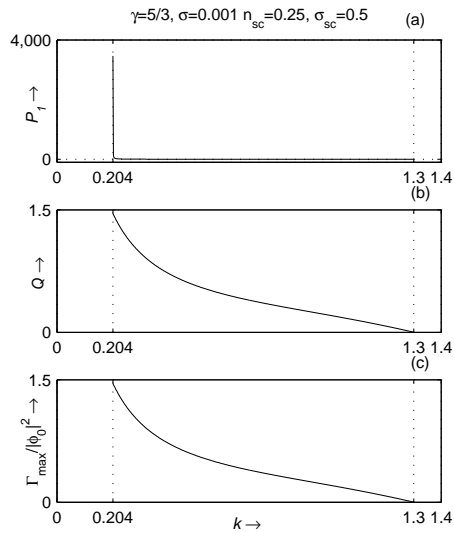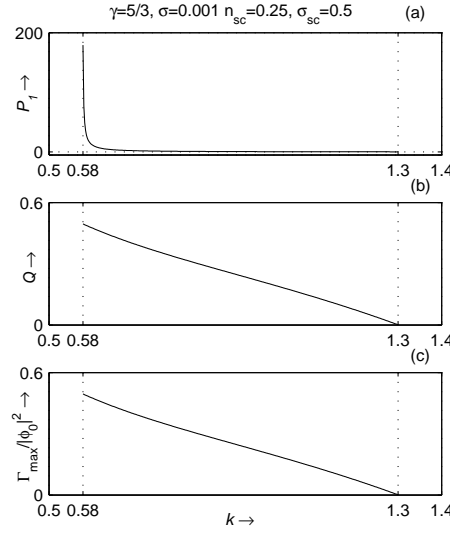
Fig. 6. P_1, Q and $\Gamma_{max}/|\phi_0|^2$ **have been plotted against** k **in (a), (b) and (c) respectively for** $n_1 = 0.25$ **and** $\omega_c = 0.5$**.**

It is simple to check that $P_1 Q$ is a function of k, n_{sc}, σ_{sc}, ω_c, n_1, σ and γ. Thus, we can say that $P_1 Q$ is a function of k and n_1 for fixed values of n_{sc}, σ_{sc}, ω_c, σ, γ and consequently, $P_1 Q = 0$ gives a functional relation between k and n_1. This functional relation between k and n_1 is plotted in figure 1 and figure 2 when $P_1 Q = 0$ and the other values of the parameters as mentioned in figure caption.

In figure 1, n_1 has been plotted against k when $P_1 Q = 0$ for different values of ω_c viz., (a) for $\omega_c = 0.01$, (b) for $\omega_c = 0.2$, (c) for $\omega_c = 0.4$ and (d) for $\omega_c = 0.6$. Here, the other values of the parameters are $n_{sc} = 0.25$, $\sigma_{sc} = 0.25$, $\gamma = 5/3$ and $\sigma = 0.001$. In figure 1, the shaded regions are the region $P_1 Q < 0$. In the shaded regions, i.e., $P_1 Q < 0$, $\Omega^2 > 0$ and consequently, the modulated IA wave is stable for any point (k, n_1) lies within the shaded regions of the figure 1. Again, the regions $P_1 Q > 0$ can be divided into two parts $K > K_c$ and $K < K_c$ where $\Omega^2 > 0$ or $\Omega^2 < 0$ according to weather $K > K_c$ or $K < K_c$. Therefore, the modulated IA wave is stable for any point (k, n_1) lies within the region $P_1 Q > 0$ with the restriction $K > K_c$

and the modulated IA wave is unstable for any point (k, n_1) lies within region $P_1 Q > 0$ with the restriction $K < K_c$. From figure 1, we see that the region $P_1 Q > 0$ decreases with increasing ω_c, i.e., instability state of the modulated IA wave decreases with increasing ω_c. In figure 1(a), we can not express n_1 as a function of k from the relation $P_1 Q = 0$, within the interval $0 < k < 0.012$. In figure 1(b), we can not express n_1 as a function of k from the relation $P_1 Q = 0$, within the interval $0 < k < 0.204$. In figure 1(c), we can not express n_1 as a function of k from the relation $P_1 Q = 0$, within the interval $0 < k < 0.432$. In figure 1(d), we can not express n_1 as a function of k from the relation $P_1 Q = 0$, within the interval $0 < k < 0.736$. Also, from figure 5, we see that the region $P_1 Q > 0$ is bounded and this region is bounded by the curves $n_1 = 1$ and $P_1 Q = 0$ whereas the region $P_1 Q < 0$ is also bounded by the curves $k = 0$, $k = 3.817$, $n_1 = 0$ and $P_1 Q = 0$.

In figure 2, n_1 has been plotted against k when $P_1 Q = 0$ for different values of ω_c viz., (a) for $\omega_c = 0.01$, (b) for $\omega_c = 0.2$, (c) for $\omega_c = 0.4$ and (d) for $\omega_c = 0.6$. Here, the other values of the

parameters are $n_{sc} = 0.25$, $\sigma_{sc} = 0.25$, $\gamma = 5/3$ and $\sigma = 0.1$. In figure 2, the regions $P_1Q > 0$ are shaded. In the region $P_1Q < 0$, $\Omega^2 > 0$ and consequently, the modulated IA wave is stable for any point (k, n_1) lies within the region $P_1Q < 0$ of figure 2. Again, all the regions $P_1Q > 0$ can also be divided into two regions, viz., $K > K_c$ and $K < K_c$ where $\Omega^2 > 0$ if $K > K_c$ and $\Omega^2 < 0$ if $K < K_c$. Therefore, the modulated IA wave is stable for any point (k, n_1) lies within the region $P_1Q > 0$ with the restriction $K > K_c$ and the modulated IA wave is unstable for any point (k, n_1) lies within the region $P_1Q > 0$ with the restriction $K < K_c$. From figure 2, we see that the region $P_1Q > 0$ decreases with increasing ω_c, i.e., instability state of the modulated IA wave decreases with increasing ω_c. In figure 2(a), we can not express n_1 as a function of k from the relation $P_1Q = 0$, within the interval $0 < k < 0.008$. In figure 2(b), we can not express n_1 as a function of k from the relation $P_1Q = 0$, within the interval $0 < k < 0.124$. In figure 2(c), we can not express n_1 as a function of k from the relation $P_1Q = 0$, within the interval $0 < k < 0.248$. In figure 2(d), we can not express n_1 as a function of k from the relation $P_1Q = 0$, within the interval $0 < k < 0.38$. Also, from figure 2, we see that the region $P_1Q > 0$ is bounded and this region is bounded by the curves $n_1 = 1$ and $P_1Q = 0$ whereas the region $P_1Q < 0$ is also bounded by the curves $k = 0$, $k = 1.844$, $n_1 = 0$ and $P_1Q = 0$.

Again, considering the figure 1 and figure 2 we see that if we increase the value of σ then we see that the instability state is decreasing and the stability regions are increasing.

P_1, Q and $\Gamma_{max}/|\phi_0|^2$ have been plotted against k in figure 3(a), figure 3(b) and figure 3(c) respectively for $\gamma = 5/3$, $\sigma = 0.001$, $n_{sc} = 0.25$, $\sigma_{sc} = 0.5$, $n_1 = 0.7$ and $\omega_c = 0.2$. Again, P_1, Q and $\Gamma_{max}/|\phi_0|^2$ have been plotted against k in figure 5(a), figure 5(b) and figure 5(c) respectively for $\gamma = 5/3$, $\sigma = 0.001$, $n_{sc} = 0.25$, $\sigma_{sc} = 0.5$, $n_1 = 0.7$ and $\omega_c = 0.5$. Clearly, we have seen that figure 3 and figure 4 are characteristically different. If $\omega_c \in (0, 0.405)$ then we have figures look like figure 4 and if $0.406 < \omega_c$ the we have figures look like figure 4. In the interval $0.84 < k < 1.3$ of figure 4, $P_1Q < 0$, i.e., the modulated IA wave is stable in the interval of k consequently,

we get a jump type of discontinuity. So, we have seen that $\Gamma_{max}/|\phi_0|^2$ decreases with increasing ω_c.

P_1, Q and $\Gamma_{max}/|\phi_0|^2$ have been plotted against k in figure 5(a), figure 5(b) and figure 5(c) respectively for $\gamma = 5/3$, $\sigma = 0.001$, $n_{sc} = 0.25$, $\sigma_{sc} = 0.5$, $n_1 = 0.25$ and $\omega_c = 0.2$. And again, P_1, Q and $\Gamma_{max}/|\phi_0|^2$ have been plotted against k in figure 6(a), figure 6(b) and figure 6(c) respectively for $\gamma = 5/3$, $\sigma = 0.001$, $n_{sc} = 0.25$, $\sigma_{sc} = 0.5$, $n_1 = 0.25$ and $\omega_c = 0.5$. Finally, from figure 5 and figure 6 we have seen that $\Gamma_{max}/|\phi_0|^2$ decreases with increasing ω_c.

6 CONCLUSIONS

We have studied the MI of IA waves in a collisionless magnetized plasmas composed of adiabatic warm ions, Maxwell-Boltzmann distribution of hot electrons as well as Maxwell-Boltzmann distribution of cold electrons. Here also we have considered the external uniform static magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) propagating along the $z-$axis. We have derived a NLSE using RPM [34, 33]. Finally, we have observed the following results:

1. Analytically we have derived the instability condition and the maximum growth rate of instability (Γ_{max}) of the modulated IA waves.

2. Numerically we have investigated the instability condition as well as Γ_{max} with respect to the different parameters n_{sc}, σ_{sc}, σ, ω_c and n_1.

3. Numerically, we have seen that the phase velocity ($\frac{\omega}{k}$) as well as the group velocity ($V_g = \frac{\partial\omega}{\partial k}$) both are decreasing functions of k for any feasible value of the parameters. For a fixed value of $k = 4$, both the phase velocity as well as the group velocity of the IA wave takes a very small numerical value.

4. $P_1Q < 0 \implies \Omega^2 > 0$. Therefore, the IA wave is modulationally stable for all $P_1Q < 0$. On the other hand, if $P_1Q > 0$, then $\Omega^2 \geq 0$ or $\Omega^2 < 0$ according to whether $K \geq K_c$ or $K < K_c$. Therefore, the IA wave is modulationally stable for $P_1Q > 0$

and with the restriction $K \geq K_c$ and the IA wave is modulationally unstable for $P_1Q > 0$ and with the restriction $K < K_c$.

5. We have seen that the region $P_1Q > 0$ decreases with increasing ω_c, i.e., instability state of the IA waves is decreases with increasing ω_c. Also, if we increase the value of σ, i.e. if we increase the ion temperature then we have seen that the instability state is decreasing and the stability regions are increasing.

6. We have seen that the region of existence of $\Gamma_{max}/|\phi_0|^2$ decreases with increasing ω_c. As well as $\Gamma_{max}/|\phi_0|^2$ decreases with increasing ω_c. Finally, we have seen that the region of existence of $\Gamma_{max}/|\phi_0|^2$ increases with increasing n_1.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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