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Peristaltic Pumping and Dispersion of a MHD Couple Stress Fluid with Chemical Reaction and Wall Effects

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Authors' contributions

This work was carried out in collaboration between both authors. Author GCS formulated the problem, and managed literature searches. Author MYD solved the problem analytically, and wrote the first draft of the manuscript. Author GCS managed the analyses of the study. Both authors read and approved the final manuscript.

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Abstract

The dispersion of a solute matter in the magneto-hydrodynamic peristaltic pumping of an incompressible couple stress fluid with wall effects has been studied. The mean effective coefficient of dispersion on simultaneous homogeneous, heterogeneous chemical reaction has been obtained through long wavelength assumption and condition of Taylor's limit. The impacts of penetrating parameters on the mean effective dispersion coefficient have been examined through the graphs. It is found that wall constraints and amplitude ratio favor the scattering, while couple stress and magnetic field constraints resist the scattering during pumping.

Keywords: Chemical reaction; couple stress fluid; dispersion; peristaltic transport; wall properties.

2010 Mathematics Subject Classification: 76V05; 76Z05; 76W99; 92C10.

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1 Introduction

Peristalsis of non-Newtonian liquids has received more attention in recent years in physiological sciences and engineering. In the fluid mechanics point of view, peristaltic creeping is described by dynamic interaction of liquid stream by the movement of stretchy boundaries. In this connection it falls in the field of moving boundary problems in applied mathematics or FSI problems in science and engineering. In the view of its importance, some workers ([1] - [3]) have investigated the peristaltic transport of various fluids under different circumstances.

Couple stress fluids are fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, such as blood, lubricants containing small amount of high polymer additive, electro-rhelogical fluids and synthetic fluids. It is seen that couple stress fluid behavior are exceptionally useful in understanding various mechanical and physiological procedures. The couple stress model introduced by Stokes [4] has distinct features. The main feature of couple stresses is to introduce a size dependent effect. These fluids are able to describe blood, suspension fluids, and various types of lubricants. Such studies clarify the behavior of rheological complex liquids. Some studies on peristaltic transport of couple stress fluid have been reported in references ([5] - [8]). The effects of wall on Poiseuille flow with peristalsis have been examined by Mittra and Prasad [9]. After this study, few investigators have explored the wall effects on different fluids with peristalsis ([10] - [14]).

Magnetohydrodynamic (MHD) peristaltic flow nature of liquid is especially imperative in physiological and mechanical procedures. In the existence of magnetic field, many fluids posses an electrically conducting nature, which is an important aspect of the physical situation in the flow problems of magnetohydrodynamics. It is useful for tumor treatment, MRI (Magnetic Resonance Imaging) scanning, blood pumping, reduction of bleeding during surgeries, targeted transportation of drugs, and so on. Magneto-therapy is an essential application to human body. This heals the diseases like ulceration, inflammations and diseases of uterus. Some researchers [15]−[17] have explored the magneto hydrodynamic character of non-Newtonian liquids through different circumstances. They discussed the effects of magnetic field, permeability, micropolar, couple stress, and wall parameters.

Dispersion of a solute describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration. Dispersion plays a crucial task in physiological systems. For example, distribution of drugs in the human body, chyme transport and other applications like environmental pollutant transportation, chromatographic separation, the mixing and transport of drugs or toxic substances in physiological structures [18]. The basic theory on dispersion was first proposed by Taylor,[19] investigated theoretically and experimentally that the dispersion of a solute is miscible with a liquid flowing through a channel. Several workers [20]−[24] have investigated the dispersion of a solute in viscous fluid, under different limitations. Furthermore, some investigators [25]−[35] extended this analysis to non Newtonian fluids.

Existing information on the topic witnessed that an analytical treatment of creeping sinusoidal flow and dispersion of a MHD couple stress fluid with chemical reaction and wallproperties has been never reported. Motivated from the reported literature, we have investigated the wall and chemical effects on the creeping sinusoidal stream and dispersion of a MHD couple stress fluid. The investigative expression for mean effective dispersion coefficient has been obtained. The effects of different values of penetrating parameters are discussed in detail through graphs.The present issue might be appropriate for the treatment on intestinal disorder, gallstones in gallbladder without surgery.

2 Formulation of the Problem

Consider the magneto-hydrodynamic couple stress fluid with peristalsis in the 2- dimensional channel. Fig. 1 depicts the wave shape.

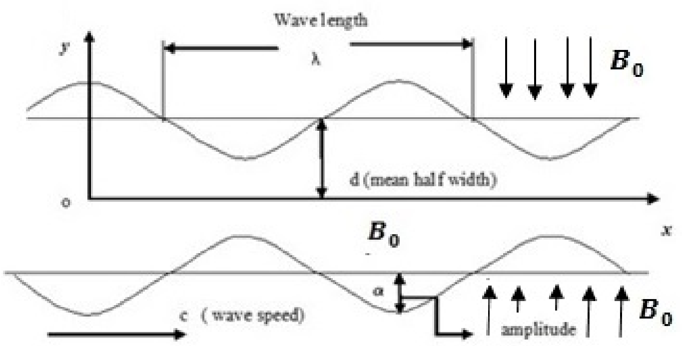

Fig. 1. Geometry of the problem

The wave shape is given by the subsequent condition ([5]):

$$\mathcal{Y} = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda} (\mathcal{X} - ct) \right], \tag{2.1}$$

where, the half width of the channel is d, the wavelength of the peristaltic wave is λ, the amplitude of the wave is a, and the wave speed is c.

The relating flow conditions (Mekheimer [15]) of the current issue are:

$$\frac{\partial \mathcal{U}}{\partial \mathcal{X}} + \frac{\partial \mathcal{V}}{\partial \mathcal{Y}} = 0, \tag{2.2}$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U} \frac{\partial}{\partial \mathcal{X}} + \mathcal{V} \frac{\partial}{\partial \mathcal{Y}} \right] \mathcal{U} = -\frac{\partial p}{\partial \mathcal{X}} + \mu \nabla^2 \mathcal{U} - \eta^{'} \nabla^4 \mathcal{U} - \sigma B_0^2 \mathcal{U}, \tag{2.3}$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U} \frac{\partial}{\partial \mathcal{X}} + \mathcal{V} \frac{\partial}{\partial \mathcal{Y}} \right] \mathcal{V} = -\frac{\partial p}{\partial \mathcal{Y}} + \mu \nabla^2 \mathcal{V} - \eta^{'} \nabla^4 \mathcal{V}, \tag{2.4}$$

where $\frac{\partial^2}{\partial \mathcal{X}^2} + \frac{\partial^2}{\partial \mathcal{Y}^2} = \nabla^2$, $\nabla^2 \nabla^2 = \nabla^4$, the constant associated with couple stress fluid is η', the fluid density is ρ, the viscosity coefficient is μ, the velocity components in the $\mathcal{X}$, $\mathcal{Y}$ direction is $\mathcal{U}$, $\mathcal{V}$, the pressure is p and the magnetic field is B_0.

Referring (Mittra-Prasad [9]), the condition of the flexible wall movement is specified as:

$$\mathcal{L}(h) = p - p_0, \tag{2.5}$$

where, the movement of the stretched membrane by the damping force is $\mathcal{L}$ and is intended by the subsequent equation:

$$\mathcal{L} = -\mathcal{T} \frac{\partial^2}{\partial \mathcal{X}^2} + m \frac{\partial^2}{\partial t^2} + \mathtt{C} \frac{\partial}{\partial t}. \tag{2.6}$$

Here, the coefficient of sticky damping force is $\mathtt{C}$, the mass per/area is m, and the membrane tension is $\mathcal{T}$.

Neglecting the body couples and body strengthens, under long - wavelength theory conditions (2.2) to (2.4) yield as:

$$\frac{\partial \mathcal{U}}{\partial \mathcal{X}} + \frac{\partial \mathcal{V}}{\partial \mathcal{Y}} = 0, \tag{2.7}$$

$$-\frac{\partial p}{\partial \mathcal{X}} + \mu \frac{\partial^2 \mathcal{U}}{\partial \mathcal{Y}^2} - \eta' \frac{\partial^4 \mathcal{U}}{\partial \mathcal{Y}^4} - \sigma B_0^2 \mathcal{U} = 0, \tag{2.8}$$

$$-\frac{\partial p}{\partial \mathcal{Y}} = 0. \tag{2.9}$$

The allied border conditions are

$$\mathcal{U} = 0, \ \ \frac{\partial^2 \mathcal{U}}{\partial \mathcal{Y}^2} = 0, \ \ \text{at} \quad \mathcal{Y} = \pm h. \tag{2.10}$$

It is presumed that $p_0 = 0$ and the channel walls are inextensible; therefore, the straight displacement of the wall is nil and only lateral movement takes place, and

$$\frac{\partial}{\partial \mathcal{X}}\mathcal{L}(h) = \mu\frac{\partial^2 \mathcal{U}}{\partial \mathcal{Y}^2} - \eta'\frac{\partial^4 \mathcal{U}}{\partial \mathcal{Y}^4} - \sigma B_0^2 \mathcal{U} = 0, \qquad \text{at} \qquad \mathcal{Y} = \pm h, \tag{2.11}$$

where

$$\frac{\partial}{\partial \mathcal{X}}\mathcal{L}(h) = \frac{\partial p}{\partial \mathcal{X}} = P' = -\mathcal{T}\frac{\partial^3 h}{\partial \mathcal{X}^3} + m\frac{\partial^3 h}{\partial \mathcal{X}\partial t^2} + \mathtt{C}\frac{\partial^2 h}{\partial \mathcal{X}\partial t}. \tag{2.12}$$

Solving the conditions (2.8) and (2.9) with (2.10) and (2.11) we obtain

$$\mathcal{U}(\mathcal{Y}) = -\frac{1}{\sigma B_0^2}\left(\frac{\partial p}{\partial \mathcal{X}}\right)\left[A_1'\cosh(m_1'\mathcal{Y}) + A_2'\cosh(m_2'\mathcal{Y}) + 1\right], \tag{2.13}$$

where, $m_1' = \sqrt{\frac{\mu}{2\eta'}\left(1+\sqrt{1-\frac{4\eta'\sigma B_0^2}{\mu^2}}\right)}$, $m_2' = \sqrt{\frac{\mu}{2\eta'}\left(1-\sqrt{1-\frac{4\eta'\sigma B_0^2}{\mu^2}}\right)}$.

The mean speed is specified as:

$$\bar{\mathcal{U}} = \frac{1}{2h}\int_{-h}^{h}\mathcal{U}(\mathcal{Y})d\mathcal{Y}. \tag{2.14}$$

Conditions (2.13) and (2.14) yield as:

$$\bar{\mathcal{U}} = -\frac{1}{\sigma B_0^2}\left(\frac{\partial p}{\partial \mathcal{X}}\right)\left[\frac{A_1'}{m_1'h}\sinh(m_1'h) + \frac{A_2'}{m_2'h}\sinh(m_2'h) + 1\right]. \tag{2.15}$$

Utilizing Ravikiran-Radhakrishnamacharya [30], the liquid speed is given by the condition:

$$\mathcal{U}_{\mathcal{X}} = \mathcal{U} - \bar{\mathcal{U}}. \tag{2.16}$$

Conditions (2.13), (2.15) and (2.16) yield as:

$$\mathcal{U}_{\mathcal{X}} = -\frac{1}{\sigma B_0^2}\left(\frac{\partial p}{\partial \mathcal{X}}\right)\left[A_1'\cosh(m_1'\mathcal{Y}) + A_2'\cosh(m_2'\mathcal{Y}) - \frac{A_1'}{m_1'h}\sinh(m_1'h) - \frac{A_2'}{m_2'h}\sinh(m_2'h)\right], \tag{2.17}$$

where
$A_1' = \frac{(m_2')^2}{\left[(m_1')^2-(m_2')^2\right]\cosh(m_1'h)}$, $A_2' = \frac{-(m_1')^2}{\left[(m_1')^2-(m_2')^2\right]\cosh(m_2'h)}$,
$P' = -\mathcal{T}\frac{\partial^3 h}{\partial \mathcal{X}^3} + m\frac{\partial^3 h}{\partial \mathcal{X}\partial t^2} + \mathtt{C}\frac{\partial^2 h}{\partial \mathcal{X}\partial t}$.

2.1 Heterogeneous-homogeneous chemical reactions with diffusion

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic pumping of a couple stress fluid in a channel under isothermal conditions. Alluding Taylor [19] and Gupta-Gupta [22], the diffusion equation for the concentration $\mathcal{C}$ of the material for the current issue is

$$\frac{\partial \mathcal{C}}{\partial t} + \mathcal{U}\frac{\partial \mathcal{C}}{\partial \mathcal{X}} = \mathcal{D}\frac{\partial^2 \mathcal{C}}{\partial \mathcal{Y}^2} - k_1\mathcal{C}. \tag{2.18}$$

Here, the rate constant of first order chemical response is k_1, the molecular diffusion coefficient is $\mathcal{D}$ and liquid concentration is $\mathcal{C}$.

The dimensionless quantities are specified as:

$$\eta = \frac{\mathcal{Y}}{d}, \xi = \frac{(\mathcal{X} - \bar{\mathcal{U}}t)}{\lambda}, \mathcal{H} = \frac{h}{d}, \mathcal{P} = \frac{d^2}{\mu c \lambda} P', \theta = \frac{t}{\bar{t}}, \bar{t} = \frac{\lambda}{\bar{\mathcal{U}}}, M = \sqrt{\frac{\sigma B_0^2 d^2}{\mu}}. \tag{2.19}$$

For the regular estimations of physiologically essential parameters of this issue, it is normal that $\bar{\mathcal{U}} \approx \mathcal{C}$ (Ravikiran-Radhakrishnamacharya [30]).

To proceed further, we use $\bar{\mathcal{U}} \approx \mathcal{C}$, in condition(2.18) and the conditions (2.12), (2.17), (2.18) are nondimensionalized as:

$$\mathcal{P} = -\epsilon \left[-E_3 (2\pi)^2 \sin(2\pi\xi) + (E_1 + E_2)(2\pi)^3 \cos(2\pi\xi) \right], \tag{2.20}$$

$$\mathcal{U}_{\mathcal{X}} = -\frac{1}{\sigma B_0^2} \frac{\partial p}{\partial \mathcal{X}} \left[A_1 \cosh(m_1 \eta) + A_2 \cosh(m_2 \eta) + A_3 \right], \tag{2.21}$$

$$\frac{\partial^2 \mathcal{C}}{\partial \eta^2} - \frac{k_1 d^2}{\mathcal{D}} \mathcal{C} = \frac{d^2}{\lambda \mathcal{D}} \mathcal{U}_{\mathcal{X}} \frac{\partial \mathcal{C}}{\partial \xi}, \tag{2.22}$$

where $m_1 = m_1' d = \sqrt{\frac{\gamma^2}{2}\left(1 + \sqrt{1 - \frac{4M^2}{\gamma^2}}\right)}$, $m_2 = m_2' d = \sqrt{\frac{\gamma^2}{2}\left(1 - \sqrt{1 - \frac{4M^2}{\gamma^2}}\right)}$,
the amplitude ratio is $\epsilon \left(= \frac{a}{d}\right)$, the rigidity is $E_1 \left(= -\frac{\mathcal{T} d^3}{\lambda^3 \mu c}\right)$, the stiffness is $E_2 = \left(\frac{mcd^3}{\lambda^3 \mu}\right)$,
the viscous damping force in the wall is $E_3 = \left(\frac{\mathsf{c} d^3}{\mu \lambda^2}\right)$, the couple stress constraint is $\gamma \left(= d\sqrt{\frac{\mu}{\eta'}}\right)$
and the magnitic field constraint is $M \left(= B_0 d \sqrt{\frac{\sigma}{\mu}}\right)$.

The dispersion with first- order irreversible chemical response occur in the mass of the liquid and at the channel walls. Referring Chandra-Phlip [26], the wall conditions are specified as

$$\frac{\partial \mathcal{C}}{\partial \mathcal{Y}} + f\mathcal{C} = 0 \quad \text{at} \quad \mathcal{Y} = h = [a \sin \frac{2\pi}{\lambda}(\mathcal{X} - \bar{\mathcal{U}}t) + d], \tag{2.23}$$

$$\frac{\partial \mathcal{C}}{\partial \mathcal{Y}} - f\mathcal{C} = 0 \quad \text{at} \quad \mathcal{Y} = -h = -[a \sin \frac{2\pi}{\lambda}(\mathcal{X} - \bar{\mathcal{U}}t) + d]. \tag{2.24}$$

Condition (2.19), (2.23) and (2.24) yields as:

$$\frac{\partial \mathcal{C}}{\partial \eta} + \beta \mathcal{C} = 0 \quad \text{at} \quad \eta = \mathcal{H} = [\epsilon \sin(2\pi\xi) + 1], \tag{2.25}$$

$$\frac{\partial \mathcal{C}}{\partial \eta} - \beta \mathcal{C} = 0 \quad \text{at} \quad \eta = -\mathcal{H} = -[\epsilon \sin(2\pi\xi) + 1], \tag{2.26}$$

where the heterogeneous response rate constraint is $\beta = \mathcal{F} d$, relating to catalytic response at the dividers.

Utilizing conditions (2.25) and (2.26), the primitive of (2.22) is obtained as:

$$\mathcal{C}(\eta) = -\frac{d^2}{\lambda \mathcal{D}} \frac{1}{\sigma B_0^2} \frac{\partial \mathcal{C}}{\partial \xi} \frac{\partial p}{\partial \mathcal{X}} \Big[A_4 \cosh(m_1 \eta) + A_5 \cosh(m_2 \eta) + A_6 \cosh(\alpha \eta) + A_7 \Big]. \tag{2.27}$$

The volumetric flow rate $\mathscr{Q}$ is specified as

$$\mathscr{Q} = \int_{-\mathcal{H}}^{\mathcal{H}} \mathcal{C}\mathcal{U}_{\mathcal{X}} d\eta. \tag{2.28}$$

Using conditions (2.21) and (2.27) in (2.28), we obtain

$$\mathscr{Q} = -2\frac{d^6}{\lambda\mu^2\mathcal{D}}\frac{\partial\mathcal{C}}{\partial\xi}G(\xi,\epsilon,\alpha,\beta,E_1,E_2,E_3,M,\gamma), \tag{2.29}$$

where

$$G(\xi,\epsilon,\alpha,\beta,E_1,E_2,E_3,M,\gamma) = -\frac{P^2}{M^4}\Big[\tfrac{A_1A_4}{2}B_1 + \tfrac{A_2A_5}{2}B_2 + (A_1A_5 + A_2A_4)B_3 + A_1A_6B_4$$
$$+ A_2A_6B_5 + (A_1A_7 + A_3A_4)B_6 + (A_2A_7 + A_3A_5)B_7 + A_3A_6B_8 + A_3A_7\mathcal{H}\Big],$$

$$A_1 = \frac{(m_2)^2}{\big[(m_1)^2-(m_2)^2\big]\cosh(m_1\mathcal{H})}, \quad A_2 = \frac{-(m_1)^2}{\big[(m_1)^2-(m_2)^2\big]\cosh(m_2\mathcal{H})},$$
$$A_3 = \frac{-(m_2)^2\sinh(m_1\mathcal{H})}{m_1\mathcal{H}\big[(m_1)^2-(m_2)^2\big]\cosh(m_1\mathcal{H})} + \frac{(m_1)^2\sinh(m_2\mathcal{H})}{m_2\mathcal{H}\big[(m_1)^2-(m_2)^2\big]\cosh(m_2\mathcal{H})},$$
$$A_4 = \frac{(m_2)^2}{\big[(m_1)^2-(\alpha)^2\big]\big[(m_1)^2-(m_2)^2\big]\cosh(m_1\mathcal{H})}, \quad A_6 = A_3L_1 - A_4L_2 - A_5L_3,$$
$$A_5 = \frac{-(m_1)^2}{\big[(m_2)^2-(\alpha)^2\big]\big[(m_1)^2-(m_2)^2\big]\cosh(m_2\mathcal{H})}, \qquad A_7 = -\frac{A_3}{\alpha^2},$$
$$L_1 = \frac{\beta}{\alpha^2(\alpha\sinh(\alpha\mathcal{H})+\beta\cosh(\alpha\mathcal{H}))}, \qquad L_2 = \frac{(m_1\sinh(m_1\mathcal{H})+\beta\cosh(m_1\mathcal{H}))}{(\alpha\sinh(\alpha\mathcal{H})+\beta\cosh(\alpha\mathcal{H}))},$$
$$L_3 = \frac{(m_2\sinh(m_2\mathcal{H})+\beta\cosh(m_2\mathcal{H}))}{(\alpha\sinh(\alpha\mathcal{H})+\beta\cosh(\alpha\mathcal{H}))}, \qquad B_1 = \frac{2m_1\mathcal{H}+\sinh(2m_1\mathcal{H})}{2m_1},$$
$$B_2 = \frac{2m_2\mathcal{H}+\sinh(2m_2\mathcal{H})}{2m_2}, \quad B_6 = \frac{\sinh(m_1\mathcal{H})}{m_1}, \quad B_7 = \frac{\sinh(m_2\mathcal{H})}{m_2}, \quad B_8 = \frac{\sinh(\alpha\mathcal{H})}{\alpha},$$
$$B_3 = \frac{m_1\sinh(m_1\mathcal{H})\cosh(m_2\mathcal{H})-m_2\cosh(m_1\mathcal{H})\sinh(m_2\mathcal{H})}{\big[(m_1)^2-(m_2)^2\big]},$$
$$B_4 = \frac{m_1\sinh(m_1\mathcal{H})\cosh(\alpha\mathcal{H})-\alpha\cosh(m_1\mathcal{H})\sinh(\alpha\mathcal{H})}{\big[(m_1)^2-(\alpha)^2\big]},$$
$$B_5 = \frac{m_2\sinh(m_2\mathcal{H})\cosh(\alpha\mathcal{H})-\alpha\cosh(m_2\mathcal{H})\sinh(\alpha\mathcal{H})}{\big[(m_2)^2-(\alpha)^2\big]}, \qquad \alpha = \sqrt{\frac{k_1}{\mathcal{D}}}d.$$

Looking at condition (2.29) with Fick's law of scattering, the dispersing coefficient $\mathcal{D}^*$ was computed to such an extent that the solute disperses near to the plane moving with the typical speed of the flow and is specified as

$$\mathcal{D}^* = 2\frac{d^6}{\mu^2\mathcal{D}}G(\xi,\epsilon,\alpha,\beta,E_1,E_2,E_3,M,\gamma). \tag{2.30}$$

The mean of G is $\bar{G}$ and is attained as

$$\bar{G} = \int_0^1 G(\xi,\epsilon,\alpha,\beta,E_1,E_2,E_3,M,\gamma)d\xi. \tag{2.31}$$

3 Outcomes and Discussion

The expression for $\bar{G}(\xi,\epsilon,\alpha,\beta,E_1,E_2,E_3,M,\gamma)$ as shown in equation (2.31) has been obtained by numerical integration using the software MATHEMATICA and the domino effects are presented through graphs. It may ensure that E_1, E_2 and E_3 cannot be zero all together.

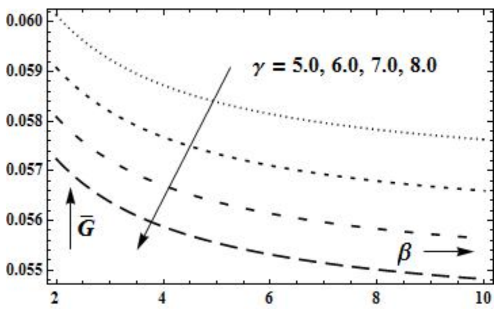

Fig. 2. Illustration of $\bar{G}$ for γ when $\epsilon = 0.2,\ \alpha = 1.0,\ M = 5.5,\ E_1 = 0.1,\ E_2 = 0.0,\ E_3 = 0.06$

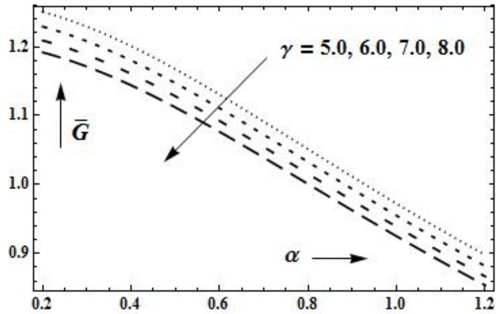

Fig. 3. Illustration of $\bar{G}$ for γ when $\epsilon = 0.2,\ \beta = 5.0,\ M = 5.5,\ E_1 = 0.1,\ E_2 = 4.0,\ E_3 = 0.06$

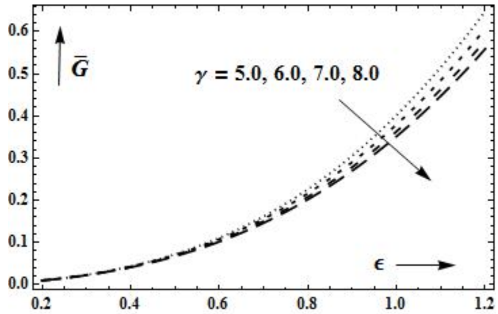

Fig. 4. Illustration of $\bar{G}$ for γ when $\alpha = 1.0,\ \beta = 5.0,\ M = 5.5,\ E_1 = 0.1,\ E_2 = 4.0,\ E_3 = 0.00$

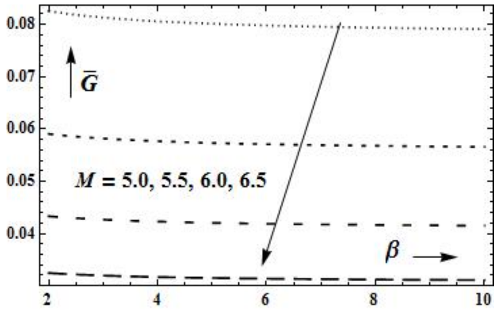

Fig. 5. Illustration of $\bar{G}$ for M when $\epsilon = 0.2,\ \alpha = 1.0,\ \gamma = 6.0,\ E_1 = 0.1,\ E_2 = 0.0,\ E_3 = 0.06$

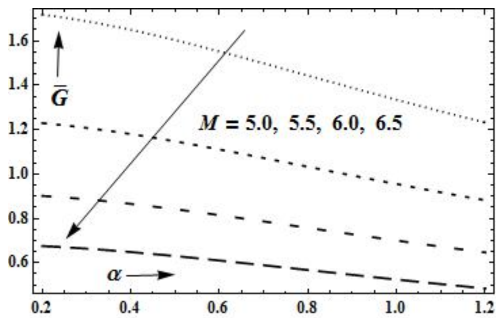

Fig. 6. Illustration of $\bar{G}$ for M when $\epsilon = 0.2,\ \beta = 5.0,\ \gamma = 6.0,\ E_1 = 0.1,\ E_2 = 4.0,\ E_3 = 0.06$

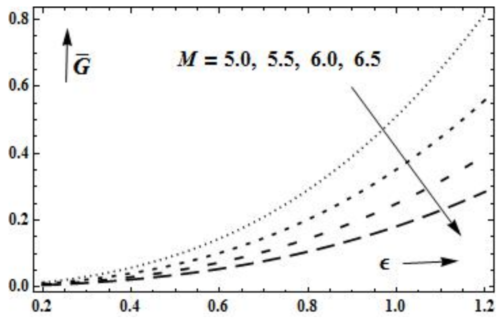

Fig. 7. Illustration of $\bar{G}$ for M when $\alpha = 1.0,\ \beta = 5.0,\ \gamma = 6.0,\ E_1 = 0.1,\ E_2 = 4.0,\ E_3 = 0.00$

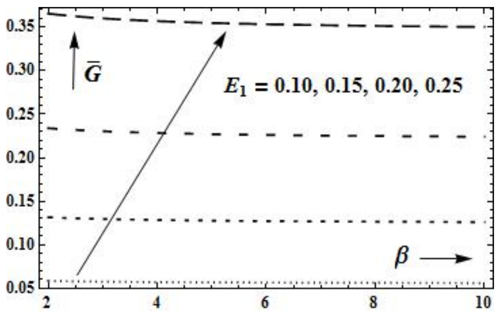

Fig. 8. Illustration of $\bar{G}$ for E_1 when $\epsilon = 0.2,\ \alpha = 1.0,\ M = 5.5,\ \gamma = 6.0,$ $E_2 = 0.0,\ E_3 = 0.0$

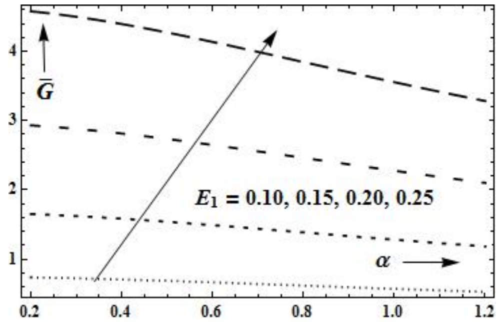

Fig. 9. Illustration of $\bar{G}$ for E_1 with $\epsilon = 0.2,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,$ $E_2 = 0.0,\ E_3 = 0.06$

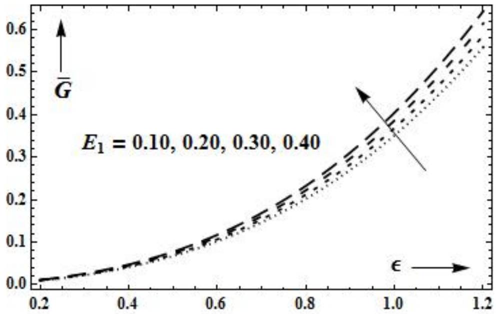

Fig. 10. Illustration of $\bar{G}$ for E_1 when $\alpha = 1.0,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,$ $E_2 = 4.0,\ E_3 = 0.00$

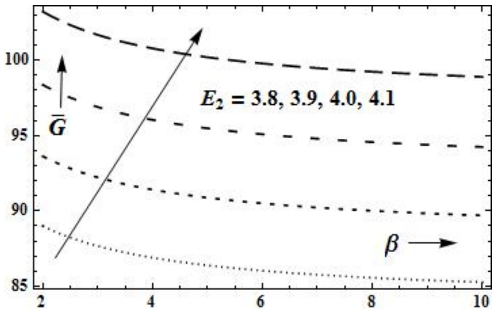

Fig. 11. Illustration of $\bar{G}$ for E_2 when $\epsilon = 0.2,\ \alpha = 1.0,\ M = 5.5,\ \gamma = 6.0,$ $E_1 = 0.1,\ E_3 = 0.00$

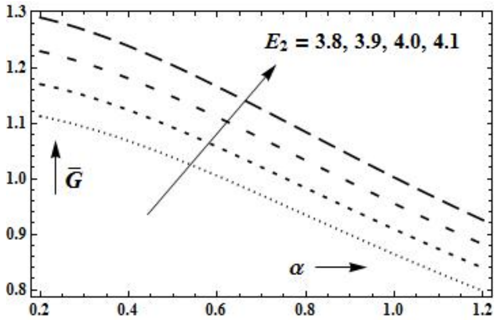

Fig. 12. Illustration of $\bar{G}$ for E_2 when $\epsilon = 0.2,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,$ $E_1 = 0.1,\ E_3 = 0.06$

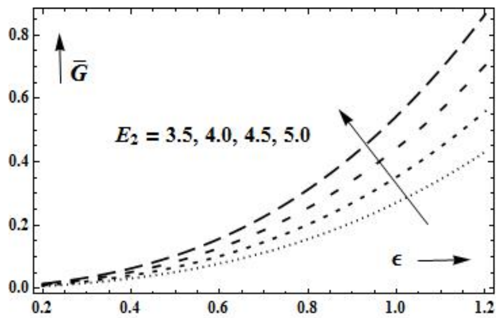

Fig. 13. Illustration of $\bar{G}$ for E_2 when $\alpha = 1.0,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,$ $E_1 = 0.1,\ E_3 = 0.06$

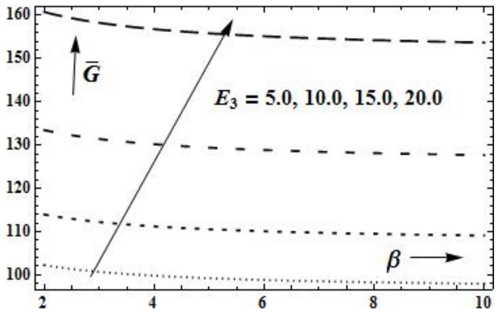

Fig. 14. Illustration of $\bar{G}$ for E_3 when $\epsilon = 0.2,\ \alpha = 1.0,\ M = 5.5,\ \gamma = 6.0,$ $E_1 = 0.1,\ E_2 = 4.0$

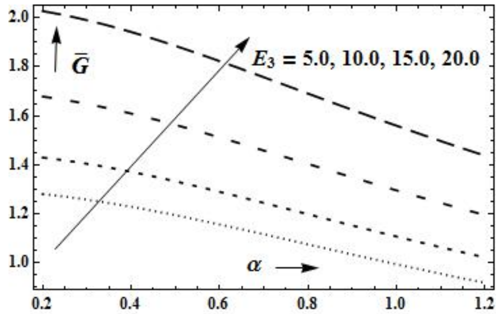

Fig. 15. Illustration of $\bar{G}$ for E_3 when $\epsilon = 0.2,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,$ $E_1 = 0.1,\ E_2 = 4.0$

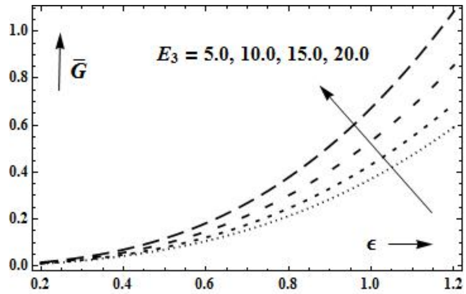

Fig. 16. Illustration of $\bar{G}$ for E_3 when $\alpha = 1.0,\ \beta = 5.0,\ M = 5.5,\ \gamma = 6.0,\ E_1 = 0.1,$ $E_2 = 4.0$

The effects of the couple stress constraint (γ) and magnetic field constraint (M) on the scattering coefficient ($\bar{G}$) are depicted in Figs. 2-7. It is observed that $\bar{G}$ descends with an increase in couple stress constraint (γ) (Figs. 2-4). Increase in couple stress constraint leads to lessen the pressure on the flow of fluid which descends the fluid velocity, as a result dispersion may reduce. This finding agrees with the conclusion of Alemayehu-Radhakrishnamacharya [28]. Figures 5-7 depicts that $\bar{G}$ descends with an increase in magnetic field constraint (M). Increase in magnetic field constraint leads to drop in the fluid velocity and as a result dispersion diminishes. This finding agrees with the conclusion of Ravikiran-Radhakrishnamacharya [30].

The impacts of the rigidity constraint (E_1) of the wall on the dissipating coefficient ($\bar{G}$) are illustrated in Figs. 8-10. It is experiential that $\bar{G}$ ascends monotonically with an expansion in E_1 in the following cases: (a) no stiffness in the wall ($E_2 = 0$) and perfectly elastic channel wall ($E_3 = 0$) (Fig. 8); (b) no stiffness in the wall ($E_2 = 0$) and dissipative wall ($E_3 \neq 0$) (Fig. 9) and (c) stiffness in the wall ($E_2 \neq 0$) and perfectly elastic wall ($E_3 = 0$) (Fig.10). It is noticed from the figures 11-13 that the mean effective dispersion coefficient increases with stiffness in the wall for the cases perfectly elastic wall ($E_3 = 0$) (Fig. 11) and dissipative wall ($E_3 \neq 0$) (Fig. 12 and 13). Figures 14 -16 shows that dispersion coefficient increases as the viscous damping force increases. This understanding might be true that increment in the flexibility of the channel walls help the stream moment which causes to enhance the dispersion. Furthermore, $\bar{G}$ ascends with an increment in the amplitude ratio (ϵ) (Figs. 4, 7, 10, 13 and 16). As already known, increment in the amplitude ratio is the expansion in the amplitude of the wave across the channel and this cause to increase the fluid velocity within the channel and consequently dispersion may enhance. This outcome concurs with that of [23] and [30].

It is seen that $\bar{G}$ descends with an increase in the homogeneous compound response rate constraint (α) (Figs. 3, 6, 9, 12, and 15). Also, it is noticed from the figures 2, 5, 8, 11, and 14 that the scattering diminishes with heterogeneous substance response rate constraint (β), and the decrease in the effective scattering coefficient is sharp in a section near to the wall. This agrees with chemical point of view because the reactions which affect diffusion happen only at the surface for heterogeneous substance response. This implies that heterogeneous substance response tends to decrease the scattering of the solute.

4 Conclusions

The effects of magnetic constraint (M), couple stress constraint (γ), amplitude ratio (ϵ), homogeneous response rate (α), heterogeneous response rate (β), rigidity (E_1), stiffness (E_2), damping characteristic (E_3) of the wall on scattering coefficient ($\bar{G}$) have been inspected for peristaltic pumping of a couple stress fluid. It is of great importance for the movement of blood in artery, bolus in esophugus, bile in bile duct and chyme in small intestine of the digestive system.

• It is seen that the concentration profile ($\bar{G}$) rises with an increase in amplitude ratio and wall constraints.

• It is noticed that concentration profile ($\bar{G}$) descends with rise in heterogeneous response rate, homogeneous response rate, couplse stress and magnetic field constraints.

• Finally, rigidity (E_1), stiffness (E_2), damping force (E_3) of the wall and amplitude ratio (ϵ) favour the dispersion, while couple stress constraint (γ) homogeneous response rate constraint (α) and heterogeneous response rate constraint (β) resist the dispersion.

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Competing Interests

Authors have declared that no competing interests exist.

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