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On Characterizations of Some Types of Fuzzy Soft Sets in Fuzzy Soft Bitopological Spaces

A. F. Sayed[1*]

[1]*Department of Mathematics, Al-Lith University College, Umm Al-Qura University, P.O. Box 112, Al-Lith 21961, Makkah Al Mukarramah, Kingdom of Saudi Arabia.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, we introduce and study the notion of kernal operator in fuzzy soft bitopological space. Moreover, some important results related to this notion are obtained. Furthermore, we introduce the concept of Alexandroff fuzzy soft topology and the concept of $\tau_1\tau_2$-Λ-fuzzy soft sets is presented and it is proved that the family of all $\tau_1\tau_2$-Λ-fuzzy soft sets is an Alexandroff fuzzy soft topology. In addition, we introduce and characterize a new type of fuzzy soft sets in a fuzzy soft bitopological spaces namely $\tau_1\tau_2$-λ-fuzzy soft closed set and investigate some of its basic properties. Finally, comparisons between these fuzzy soft sets are obtained.

Keywords: *Soft set; fuzzy set; fuzzy soft set; fuzzy soft topological space; fuzzy soft bitopological space.*

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**Corresponding author: E-mail: dr.afsayed@hotmail.com;*

1 Introduction

In the year 1965, Zadeh [1] introduced the concept of fuzzy set theory and its applications can be found in many branches of mathematical and engineering sciences including management science, control engineering, computer science, artificial intelligence (see, [2], [3]).

In the year 1999, Russian researcher Molodtsov [4], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences (see, [5], [6]). In 2003, Maji et al. [7], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D.Chen [8] , presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

In 1963, J. C. Kelly [9], first initiated the concept of bitopological spaces and other authors have contributed to development and construction some properties of such spaces (see, [10], [11]) as a generalizations of which are in general topology.

In 2014, B. M. Ittanagi [12], introduced and studied the concept of soft bitopological spaces and other authors have contributed to development and construction some properties of such spaces (see, [13], [14]).
The notion of a soft bitopological space was introduced using different soft topologies on an initial universe set. On the other hand, the mixed type of soft set theory was given using different soft topologies (see, [15], [16], [17]).

In 2015, Mukherjee and Park [18] were first introduced the notion of fuzzy soft bitopological space and they introduced the notions of $\tau_1\tau_2$-fuzzy soft open(closed) sets, $\tau_1\tau_2$-fuzzy soft interior (resp. closure) and studied some of their basic properties. In order to give the readers more information (see, [19], [20]). In this paper, we introduce and study the notion of kernal operator in fuzzy soft bitopological space. Moreover, some important results related to this notion are obtained. Furthermore,we introduce the concept of Alexandroff fuzzy soft topology and the concept of $\tau_1\tau_2$-Λ-fuzzy soft sets is presented and it is proved that the family of all $\tau_1\tau_2$-Λ-fuzzy soft sets is an Alexandroff fuzzy soft topology. In addition, we introduce and characterize a new type of fuzzy soft sets in a fuzzy soft bitopological spaces namely $\tau_1\tau_2$-λ-fuzzy soft closed set and investigate some of its basic properties. Finally, comparisons between these fuzzy soft sets are obtained.

2 Preliminaries

In this section we have presented the basic definitions and results of fuzzy soft set and fuzzy soft bitopological space which will be a central role in our paper.
Throughout our discussion, X refers to an initial universe, E the set of all parameters for X and $P(X)$ denotes the power set of X.

Definition 2.1. [1] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \to [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. Let I^X denotes the family of all fuzzy sets on X.

A member A in I^X is contained in a member B of I^X denoted $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for every $x \in X$ (see [1]).

Let $A, B \in I^X$, we have the following fuzzy sets (see [1]).

(1) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$. (Equality),

(2) $C = A \wedge B \in I^X$ by $\mu_C(x) = min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$. (Intersction),

(3) $D = A \vee B \in I^X$ by $\mu_C(x) = max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.(Union),

(4) $E = A^c \in I^X$ by $\mu_E(x) = 1 - \mu_A(x)$ for all $x \in X$. (Complement).

Definition 2.2. [1] An empty fuzzy set on X denoted by 0_X is a function which maps each $x \in X$ to 0. That is, $0_X(x) = 0$ for all $x \in X$.
A universal fuzzy set denoted by 1_X is a function which maps each $x \in X$ to 1. That is, $1_X(x) = 1$ for all $x \in X$.

Definition 2.3. [4] Let $A \subseteq E$. A pair (F, A) is called a soft set over X if F is a mapping $F : A \to P(X)$.

Definition 2.4. [21] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X, where f is a mapping given by $f : A \to I^X$ defnned by $f_A(e) = \mu^e_{f_A}$ where

$$\mu^e_{f_A} = \begin{cases} \tilde{0}, & \text{if } e \notin A; \\ otherwise, & \text{if } e \in A. \end{cases}$$

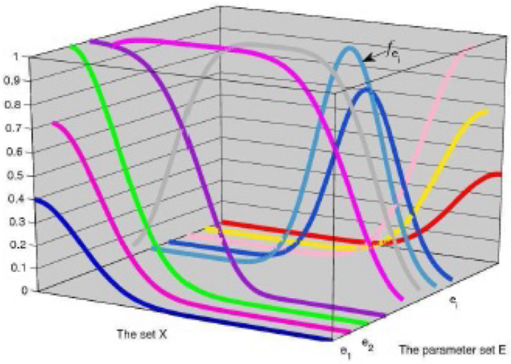

Fig. 1. A fuzzy soft set f_A

$\widetilde{(X, E)}$ denotes the family of all fuzzy soft sets over X.

Definition 2.5. [7] A fuzzy soft set $f_A \tilde{\in} \widetilde{(X, E)}$ is said to be:

(a) NULL fuzzy soft set, denoted by $\tilde{0}_E$, if for all $e \in A, f_A(e) = 0_X$.

(b) absolute fuzzy soft set, denoted by $\tilde{1}_E$, if for all $e \in E, f_A(e) = 1_X$.

Definition 2.6. [22] The complement of a fuzzy soft set (f, A), denoted by $(f, A)^c$, is defnned by $(f, A)^c = (f^c, A), f^c_A : E \to I^X$ is a mapping given by $\mu^e_{f^c_A} = 1_X - \mu^e_{f_A}$, where $1_X(x) = 1$, for all $x \in X$. Clearly $(f^c_A)^c = f_A$.

Definition 2.7. [22] Let $f_A, g_B \in \widetilde{(X, E)}$. f_A is fuzzy soft subset of g_B, denoted by $f_A \tilde{\subseteq} g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \leq \mu^e_{g_B}$ for all $e \in A$, i.e. $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x)$ for all $x \in X$ and for all $e \in A$.

Definition 2.8. [22] Let $f_A, g_B \tilde{\in} \widetilde{(X, E)}$.The union of f_A and g_B is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C, h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}$. Here we write $h_C = f_A \tilde{\cup} g_B$.

Definition 2.9. [22] Let $f_A, g_B \tilde{\in} \widetilde{(X,E)}$.The intersection of f_A and g_B is also a fuzzy soft set d_C, where $C = A \cap B$ and for all $e \in C, d_C(e) = \mu^e_{d_c} = \mu^e_{f_A} \wedge \mu^e_{g_B}$. Here we write $d_C = f_A \tilde{\cap} g_B$.

Definition 2.10. [23] The fuzzy soft set $f_A \tilde{\in} \widetilde{(X,E)}$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha(0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = 0$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.11. [23] The fuzzy soft point f_e is said to be belonging to the fuzzy soft set (g, A), denoted by $f_e \tilde{\in} (g, A)$, if for the element $e \in A$, $\alpha \leq \mu^e_{g_A}(x), (0 < \alpha \leq 1)$.

Definition 2.12. [24] Let f_A be fuzzy soft set over X. The two fuzzy soft points $f_{e_1}, f_{e_2} \tilde{\in} f_A$ are said to be equal if $\mu_{f_{e_1}}(x) = \mu_{f_{e_2}}(x)$ for all $x \in X$. Thus $f_{e_1} \neq f_{e_2}$ if and only $\mu_{f_{e_1}}(x) \neq \mu_{f_{e_2}}(x)$ for all $x \in X$.

Definition 2.13. [22] A fuzzy soft topology τ over (X, E) is a family of fuzzy soft sets over (X, E) satisfying the following properties

(i) $\tilde{0}_E, \tilde{1}_E \in \tau$

(ii) if $f_A, g_B \in \tau$, then $f_A \tilde{\cap} g_B \in \tau$,

(iii) if $f_{A_\alpha} \in \tau$ for all $\alpha \in \Delta$ an index set, then $\tilde{\bigcup}_{\alpha \in \Delta} f_{A_\alpha} \in \tau$.

Definition 2.14. [18] If τ is a fuzzy soft topology on (X, E) the triple (X, E, τ) is said to be a fuzzy soft topological space. Also each member of τ is called a fuzzy soft open set in (X, E, τ).

The complement of a fuzzy soft open set is a fuzzy soft closed set.

Definition 2.15. [18] Let (X, E, τ_1) and (X, E, τ_2) be the two different fuzzy soft topologies on (X, E). Then (X, E, τ_1, τ_2) is called a fuzzy soft bitopological space on which no seperation axioms are assumed unless explicitly stated.

The members of $\tau_i (i = 1, 2)$ are called $\tau_i (i = 1, 2)$-fuzzy soft open sets and the complement of $\tau_i (i = 1, 2)$- fuzzy soft open sets are called $\tau_i (i = 1, 2)$-fuzzy soft closed sets.

Definition 2.16. [18] A fuzzy soft set $f_E \tilde{\in} \widetilde{(X,E)}$ is called $\tau_1\tau_2$- fuzzy soft open set if $f_E = g_E \tilde{\cup} h_E$ such that $g_E \tilde{\in} \tau_1$ and $h_E \tilde{\in} \tau_2$.
The complement of $\tau_1\tau_2$- fuzzy soft open set is called $\tau_1\tau_2$- fuzzy soft closed set.

The family of all $\tau_1\tau_2$- fuzzy soft open (closed) sets in (X, E, τ_1, τ_2) is denoted by $\tau_1\tau_2 FSO(X, \tau_1, \tau_2)_E$ $(\tau_1\tau_2 FSC(X, \tau_1, \tau_2)_E)$, respectively.

Remark 2.1. [18] $\tau_i (i = 1, 2)$- fuzzy soft open is $\tau_1\tau_2$- fuzzy soft open but the converse is not true.

Theorem 2.1. *[18] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space, then $\tau = \tau_1 \tilde{\cap} \tau_2$ is a fuzzy soft topological space over (X, E).*

Remark 2.2. [18] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space, then $\tau = \tau_1 \tilde{\cup} \tau_2$ is not a fuzzy soft topological space over (X, E).

Definition 2.17. [18] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X,E)}$. Then the $\tau_i (i = 1, 2)$-fuzzy soft closure of f_E, denoted by $\tau_i cl(f_E)$, is the intersection of all $\tau_i (i = 1, 2)$-fuzzy soft closed supersets of f_E.

Clearly, $\tau_i cl(f_E)$ is the smallest $\tau_i (i = 1, 2)$- fuzzy soft closed set over (X, E) which contains f_E.

Definition 2.18. [18] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X, E)}$. Then the $\tau_1\tau_2$- fuzzy soft closure of f_E, denoted by $\tau_1\tau_2 cl(f_E)$, is the intersection of all $\tau_1\tau_2$- fuzzy soft closed supersets of f_E.

Clearly, $\tau_1\tau_2 cl(f_E)$ is the smallest $\tau_1\tau_2$- fuzzy soft closed set over (X, E) which contains f_E.

Remark 2.3. [18] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X, E)}$. Then $\tau_1\tau_2 cl(f_E) \tilde{\subseteq} \tau_i cl(f_E)(i = 1, 2)$.

Definition 2.19. [18] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X, E)}$. Then the $\tau_i(i = 1, 2)$-fuzzy soft interior of f_E, denoted by $\tau_i int(f_E)$, is the union of all $\tau_i(i = 1, 2)$ -fuzzy soft open subsets of f_E.

Clearly, $\tau_i int(f_E)$ is the largest $\tau_i(i = 1, 2)$- fuzzy soft open set over (X, E) which contained in f_E.

Definition 2.20. [18] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X, E)}$. Then the $\tau_1\tau_2$- fuzzy soft interior of f_E, denoted by $\tau_1\tau_2 int(f_E)$, is the union of all $\tau_1\tau_2$- fuzzy soft open subsets of f_E.

Clearly, $\tau_1\tau_2 cl(f_E)$ is the largest $\tau_1\tau_2$-fuzzy soft open set over (X, E) which contained in f_E.

Remark 2.4. [18] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space and $f_E \tilde{\in} \widetilde{(X, E)}$. Then $\tau_i int(f_E)(i = 1, 2) \tilde{\subseteq} \tau_1\tau_2 int(f_E)$.

3 Characterizations of Some Types of Fuzzy Soft Sets in Fuzzy Soft Bitopological Spaces

In this section, we introduce and study the notion of kernal operator in fuzzy soft bitopological space. Moreover, some important results related to this notion are obtained. Furthermore, the concept of $\tau_1\tau_2$-Λ-fuzzy soft sets is presented and it is proved that the family of all $\tau_1\tau_2$-Λ-fuzzy soft sets is an Alexandroff fuzzy soft topology. In addition, we introduce and characterize a new type of fuzzy soft sets in a fuzzy soft bitopological spaces namely $\tau_1\tau_2$-λ-fuzzy soft closed set and investigate some of its basic properties. Finally, comparisons between these fuzzy soft sets are obtained.

Definition 3.1. Let (X, E, τ_1, τ_2) be a soft bitopological space. Then, the family of all $\tau_1\tau_2$- fuzzy soft open sets is a supra fuzzy soft topology on X. This supra fuzzy soft topology, will denoted by τ_{12}, i.e., $\tau_{12} = \tau_1\tau_2 FSO(X, \tau_1, \tau_2)_E = \{g_E = g_{1_E} \tilde{\cup} g_{2_E} : g_{i_E} \in \tau_i, i = 1, 2\}$ and the triple (X, E, τ_{12}) is the supra fuzzy soft topological space associated to the fuzzy soft bitopological space (X, E, τ_1, τ_2).

Remark 3.1. (1) τ_{12} is not fuzzy soft topology in general.

(2) The finite intersection of $\tau_1\tau_2$- fuzzy soft open sets need not be a $\tau_1\tau_2$- fuzzy soft open set.

(3) The arbitrary union of $\tau_1\tau_2$- fuzzy soft closed sets need not be a $\tau_1\tau_2$- fuzzy soft closed set. These are shown in the following example:

Example 3.1. *Let* $X = \mathbb{N}, E = \{0, 1\}$ *and let* $g_{n_E} \tilde{\in} \widetilde{(\mathbb{N}, E)}$ *defined as*

$$g_{n_E} : E \to I^{\mathbb{N}},$$

such that
$g_n(0) = \tilde{0}_E$ *and* $g_n(1) = \{n/\alpha_n, n+1/\alpha_{n+1}, n+2/\alpha_{n+2}, ...\}$ *where* $\alpha_n, \alpha_{n+1}, \alpha_{n+2}, ... \in (0,1]$
and let
$\tau_1 = \{\tilde{0}_E, \tilde{1}_E\}\tilde{\bigcup}\{g_{n_E} : n = 1,2,3,...\}$.
It is clear that τ_1 *is a fuzzy soft topology on* $\mathbb{N}$.

Now, let $h_{m_E}\tilde{\in}\widetilde{(\mathbb{N}, E)}$ *defined as*

$$h_{m_E} : E \to I^{\mathbb{N}},$$

such that
$h_m(0) = \tilde{0}_E$ *and* $h_m(1) = \{/\beta_1, n+1/\beta_2, n+2/\beta_2, ...\}$ *where* $\beta_1, \beta_2, \beta_3, ... \in (0,1]$
and let
$\tau_2 = \{\tilde{0}_E, \tilde{1}_E\}\tilde{\bigcup}\{h_{m_E} : m = 1,2,3,...\}$.
Then τ_2 *is a fuzzy soft topology on* $\mathbb{N}$. *Consequently,* $(\mathbb{N}, E, \tau_1, \tau_2)$ *be a fuzzy soft bitopological space.*

Now, $g_{3_E} \in \tau_1$ *and* $h_{3_E} \in \tau_2$ *are* $\tau_1\tau_2$*-fuzzy soft open sets (for* $\tau_1, \tau_2 \subset \tau_{12}$*). But,* $g_{3_E}\tilde{\cap}h_{3_E}$ *is not* $\tau_1\tau_2$*-fuzzy soft open set, for* $g_{3_E}\tilde{\cap}h_{3_E} = f_E$ *such that* $f(0) = g_3(0) \wedge h_3(0) = 0_{\mathbb{N}}$*, where* $0_{\mathbb{N}}$ *is the empty fuzzy set on* $\mathbb{N}$.
$f(1) = g_3(1) \wedge h_3(1) = \{3/\gamma_3, 4/\gamma_4, ...\} \wedge \{1/\gamma_1, 2/\gamma_2, 3/\gamma_3\} = \{3/\gamma_3\}$ *, where* $\gamma_i = min\{\alpha_i, \beta_i\}$, $\alpha_i, \beta_i, \gamma_i \in (0,1]$*, for all* $i = 1,2,3, ...$
It is clear that f_E *cannot be expressed as a union of two fuzzy soft sets one belongs to* τ_1 *and the other belongs to* τ_2*, i.e.,* f_E *is not* $\tau_1\tau_2$*-fuzzy soft open set. Hence, the finite intersection of* $\tau_1\tau_2$*-fuzzy soft open sets need not be a* $\tau_1\tau_2$*-fuzzy soft open set. Consequently,* τ_{12} *is not fuzzy soft topology in general.*

On the other hand, since g_{3_E} *and* h_{3_E} *are* $\tau_1\tau_2$*-fuzzy soft open sets, then* $g^c_{3_E}$ *and* $h^c_{3_E}$ *are* $\tau_1\tau_2$*-fuzzy soft closed sets, but* $g^c_{3_E}\tilde{\cup}h^c_{3_E}$ *is not* $\tau_1\tau_2$*-fuzzy soft closed set,*

because $g^c_{3_E}\tilde{\cup}h^c_{3_E} = m_E$ *such that* $m(0) = g^c_3(0) \vee h^c_3(0) = 1_{\mathbb{N}}$ *(a universe fuzzy set on* $\mathbb{N}$*)*
$m(1) = g^c_3(1) \vee h^c_3(1) = 1_{\mathbb{N}} \setminus \{3/\gamma_3\}$.

It is clear that m_E *cannot be expressed as an intersection of two fuzzy soft sets one belongs to* τ^c_1 *and the other belongs to* τ^c_2*, i.e.,* m_E *is not* $\tau_1\tau_2$*-fuzzy soft closed set. Therefore, the arbitrary union of* $\tau_1\tau_2$*-fuzzy soft closed sets need not be a* $\tau_1\tau_2$*-fuzzy soft closed set.*

Definition 3.2. Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and let $g_E\tilde{\in}\widetilde{(X, E)}$. The $\tau_1\tau_2$- fuzzy soft kernel of g_E [briefly, $\tau_1\tau_2 fsker(g_E)$], is the intersection of all $\tau_1\tau_2$- fuzzy soft open supersets of g_E, i.e.,

$$\tau_1\tau_2 fsker(g_E) = \tilde{\bigcap}\{h_E \in \tau_{12} : g_E\tilde{\subset}h_E\}.$$

Remark 3.2. The $\tau_1\tau_2$- fuzzy soft kernel of g_E is not $\tau_1\tau_2$- fuzzy soft open set in general. This is shown in the following example:

Example 3.2. *Let* $X = \{x, y\}, E = \{e_1, e_2\}$ *and let*
$\tau_1 = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}\}$,
where
$g_{1_E} = \{g_1(e_1) = \{x/0.5, y/0.7\}, g_1(e_2) = \{x/0.3, y/0.6\}\}$,
$g_{2_E} = \{g_2(e_1) = \{x/0.4, y/0.5\}, g_2(e_2) = \{x/0.2, y/0.3\}\}$,
$f_{1_E} = \{f_1(e_1) = \{x/0.3, y/0.5\}, f_1(e_2) = \{x/0.3, y/0.6\}\}$,
$f_{2_E} = \{f_2(e_1) = \{x/0.4, y/0.5\}, f_2(e_2) = \{x/0.4, y/0.9\}\}$.

Then (X, E, τ_1, τ_2) *is a fuzzy soft bitopological space.*
It is clear that $\tau_{12} = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}, f_{1_E}, f_{2_E}, p_E, q_E\}$ *, where*
$p_E = g_{1_E}\tilde{\cup}f_{2_E} = \{p(e_1) = \{x/0.5, y/0.7\}, p(e_2) = \{x/0.4, y/0.9\}\}$,
$q_E = g_{2_E}\tilde{\cup}f_{1_E} = \{q(e_1) = \{x/0.4, y/0.5\}, q(e_2) = \{x/0.3, y/0.6\}\}$ *, while*
$g_{1_E}\tilde{\cup}f_{1_E} = f_{1_E}, g_{2_E}\tilde{\cup}f_{2_E} = f_{2_E}$
Now, let
$g_E = \{g(e_1) = \{x/0.2, y/0.3\}, g(e_2) = \{x/0.1, y/0.4\}\}$,
Then, the $\tau_1\tau_2$*- fuzzy soft open sets which contains* g_E *are* $\tilde{1}_E, g_{1_E}, f_{1_E}, f_{2_E}, p_E$.
Therfore,
$\tau_1\tau_2 fsker(g_E) = \tilde{1}_E\tilde{\cap}p_E\tilde{\cap}g_{1_E}\tilde{\cap}f_{1_E}\tilde{\cap}f_{2_E} = f_{1_E}$ *is a* $\tau_1\tau_2$*- fuzzy soft open set.*
But if
$m_E = \{m(e_1) = \{x/0.3, y/0.5\}, m(e_2) = \{x/0.2, y/0.3\}\}$,
Then, the $\tau_1\tau_2$*- fuzzy soft open sets which contains* m_E *are* $\tilde{1}_E, g_{1_E}, g_{2_E}, f_{1_E}, f_{2_E}, p_E, g_E$.
Therfore,
$\tau_1\tau_2 fsker(m_E) = \tilde{1}_E\tilde{\cap}g_{1_E}, g_{2_E}\tilde{\cap}f_{1_E}\tilde{\cap}f_{2_E}\tilde{\cap}p_E, g_E = m_E$, *is not a* $\tau_1\tau_2$*- fuzzy soft open set.*
Let $h_E = g_{1_E}$.
Then, the $\tau_1\tau_2$*- fuzzy soft open sets which contains* h_E *are* $\tilde{1}_E, g_{1_E}$.
Therfore,
$\tau_1\tau_2 fsker(h_E) = \tilde{1}_E\tilde{\cap}g_{1_E} = h_E$.

Theorem 3.3. *Let* (X, E, τ_1, τ_2) *be a fuzzy soft bitopological space and let* $g_E, h_E\tilde{\in}\widetilde{(X, E)}$. *Then,*

(1) $\tau_1\tau_2 fsker(\tilde{1}_E) = \tilde{1}_E$ *and* $\tau_1\tau_2 fsker(\tilde{0}_E) = \tilde{0}_E$.

(2) $g_E\tilde{\subseteq}\tau_1\tau_2 fsker(g_E)$.

(3) $g_E\tilde{\subseteq}h_E \Rightarrow \tau_1\tau_2 fsker(g_E)\tilde{\subseteq}\tau_1\tau_2 fsker(h_E)$.

(4) *If* $g_E \in \tau_{12}$, *then* $\tau_1\tau_2 fsker(g_E) = g_E$.

(5) $\tau_1\tau_2 fsker[\tau_1\tau_2 fsker(g_E)] = \tau_1\tau_2 fsker(g_E)$.

(6) $\tau_1\tau_2 fsker[\tilde{\cap}(h_{i_E}) : i \in \Delta]\tilde{\subseteq}\tilde{\bigcap}\{\tau_1\tau_2 fsker(h_{i_E}) : i \in \Delta\}$.

(7) $\tau_1\tau_2 fsker[\tilde{\cup}(g_{i_E}) : i \in \Delta] = \tilde{\bigcup}\{\tau_1\tau_2 fsker(g_{i_E}) : i \in \Delta\}$.

Proof. $(1), (2), (3)$ and (4) are obvious.

(5) Since, $g_E \in \tau_1\tau_2 fsker(g_E)$, then $\tau_1\tau_2 fsker(g_E)\tilde{\subseteq}\tau_1\tau_2 fsker[\tau_1\tau_2 fsker(g_E)]$.
Now, since $\tau_1\tau_2 fsker(g_E) = \tilde{\bigcap}\{h_E \in \tau_{12} : g_E\tilde{\subseteq}h_E\}$,
then $\tau_1\tau_2 fsker(g_E)\tilde{\subseteq}h_E$ for all $h_E \in \tau_{12}, g_E\tilde{\subseteq}h_E$
$\Rightarrow \tau_1\tau_2 fsker[\tau_1\tau_2 fsker(g_E)]\tilde{\subseteq}\tau_1\tau_2 fsker(h_E)$ for all $h_E \in \tau_{12}, g_E\tilde{\subseteq}h_E$
$\Rightarrow \tau_1\tau_2 fsker[\tau_1\tau_2 fsker(g_E)]\tilde{\subseteq}h_E$ (by (4)) for all $h_E \in \tau_{12}, g_E\tilde{\subseteq}h_E$
$\Rightarrow \tau_1\tau_2 fsker[\tau_1\tau_2 fsker(g_E)]\tilde{\subseteq}\tilde{\bigcap}\{h_E \in \tau_{12}, g_E\tilde{\subseteq}h_E\} = \tau_1\tau_2 fsker(g_E)$.
Hence, (5) holds.

(6) Since, $\tilde{\bigcap}\{h_{i_E} : i \in \Delta\}\tilde{\subseteq}h_{i_E}$ for all $i \in \Delta$, then by(3)
$\tau_1\tau_2 fsker[\tilde{\bigcap}\{h_{i_E} : i \in \Delta\}]\tilde{\subseteq}\tau_1\tau_2 fsker(h_{i_E})$ for all $i \in \Delta$. Therefore,
$\tau_1\tau_2 fsker[\tilde{\bigcap}\{h_{i_E} : i \in \Delta\}]\tilde{\subseteq}\tilde{\bigcap}\{\tau_1\tau_2 fsker(h_{i_E}) : i \in \Delta\}$

(7) Since, $g_{i_E}\tilde{\subseteq}\tilde{\bigcup}\{g_{i_E} : i \in \Delta\}$, then $\tau_1\tau_2 fsker(g_{i_E})\tilde{\subseteq}\tau_1\tau_2 fsker[\tilde{\cup}(g_{i_E}) : i \in \Delta]$
$\Rightarrow \tilde{\bigcup}\{\tau_1\tau_2 fsker(g_{i_E}) : i \in \Delta\}\tilde{\subseteq}\tau_1\tau_2 fsker[\tilde{\cup}\{g_{i_E}) : i \in \Delta\}]$.

To prove the inverse inclusion, let $f_e\tilde{\notin}\tilde{\bigcup}\{\tau_1\tau_2 fsker(g_{i_E}) : i \in \Delta\}$.
Then $f_e\tilde{\notin}\tau_1\tau_2 fsker(g_{i_E})$ for all $i \in \Delta$. Therefore, for all $i \in \Delta$ there exists $h_{i_E} \in \tau_{12}$ such that

$g_{i_E} \tilde{\subseteq} h_{i_E}$ and $f_e \tilde{\notin} h_{i_E}$. We set $h_E = \tilde{\bigcup}\{h_{i_E} : i \in \Delta\}$.
Hence $h_E \in \tau_{12}$ and $f_e \tilde{\notin} h_E$.
Since, $\tilde{\bigcup}\{g_{i_E} : i \in \Delta\} \tilde{\subseteq} \tilde{\bigcup}\{h_{i_E} : i \in \Delta\}$, then $\tilde{\bigcup}\{g_{i_E}) : i \in \Delta\} \tilde{\subseteq} h_E, h_E \in \tau_{12}$ and $f_e \tilde{\notin} h_E$.
Therefore,$f_e \tilde{\notin} \tau_1\tau_2 fsker[\tilde{\cup}(g_{i_E}) : i \in \Delta]$. Hence,(7)holds. □

Theorem 3.4. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and let $g_E, h_E \tilde{\in} \widetilde{(X, E)}$. Then,*
$f_e \tilde{\in} \tau_1\tau_2 fsker(g_E) \Leftrightarrow (s(f_e))_E \tilde{\cap} g_E \neq \tilde{0}_E$ *for all* $(s(f_e))_E \in \tau_{12}^c(f_e)$,
where $(s(f_e))_E$ is any $\tau_1\tau_2$- fuzzy soft closed set contains f_e and $\tau_{12}^c(f_e)$ is the family of all $\tau_1\tau_2$-fuzzy soft closed sets contains f_e

Proof. Let $f_e \tilde{\in} \tau_1\tau_2 fsker(g_E)$ and assume that there exists $(s(f_e))_E \in \tau_{12}^c(f_e)$ such that $(s(f_e))_E \tilde{\cap} g_E = \tilde{0}_E$. Then $g_E \tilde{\subseteq} ((s(f_e))_E)^c$ implies $\tau_1\tau_2 fsker(g_E) \tilde{\subseteq} \tau_1\tau_2 fsker(((s(f_e))_E)^c) = ((s(f_e))_E)^c$ which implies $\tau_1\tau_2 fsker(g_E) \tilde{\cap} (s(f_e))_E = \tilde{0}_E$, a contradiction.

Conversely, suppose that $f_e \tilde{\notin} \tau_1\tau_2 fsker(g_E)$. Then, there exists $h_E \in \tau_{12}$ such that $g_E \tilde{\subseteq} h_E$ and $f_e \tilde{\notin} h_E$, therefore $f_e \tilde{\in} h_E^c$, but $h_E^c \in \tau_{12}^c(f_e)$, so, by hypothesis, $h_E^c \tilde{\cap} g_E \neq \tilde{0}_E$ which contradicts with $g_E \tilde{\subseteq} h_E$. Thus, $f_e \tilde{\in} \tau_1\tau_2 fsker(g_E)$ □

Definition 3.3. A fuzzy soft set $g_E \in \widetilde{(X, E)}$ is said to be a $\tau_1\tau_2 - \Lambda$-fuzzy soft set in a fuzzy soft bitopological space (X, E, τ_1, τ_2) if $\tau_1\tau_2 fsker(g_E) = g_E$. The family of all $\tau_1\tau_2$-Λ-fuzzy soft sets will be denoted by $\tau_1\tau_2$-Λ-$FSS(X, \tau_1, \tau_2)_E$

Example 3.5. *As in Example 3.2. Let $h_E = g_{1_E}$. It is clear that h_E is $\tau_1\tau_2$-Λ-fuzzy soft set.*

Theorem 3.6. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. If g_E is $\tau_1\tau_2$-fuzzy soft set, then it is a $\tau_1\tau_2$-Λ-fuzzy soft set i.e.*
$\tau_{12} \subseteq \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$.

Proof. Immediate from the Theorem 3.3 and Definition 3.3. □

The following Example shows the converse of the above theorem is not true in general.

Example 3.7. *As in Example 3.2. Let $g_E = g_{1_E}$. It is clear that h_E is $\tau_1\tau_2 - \Lambda$-fuzzy soft set.*
It is clear that $\tau_1\tau_2 fsker(m_E) = m_E \notin \tau_{12}$.
Therefore m_E is $\tau_1\tau_2$-Λ-fuzzy soft set, but it is not a $\tau_1\tau_2$-fuzzy soft open set.

Theorem 3.8. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space.*
Then, the class $\tau_1\tau_2$-Λ-$FSS(X, \tau_1, \tau_2)_E$ is a fuzzy soft topology on X. This fuzzy soft topology, will be denoted by $\tau_{12\Lambda}$. The triple $(X, E, \tau_{12\Lambda})$ is the fuzzy soft topological space associated to the fuzzy soft bitopological space (X, E, τ_1, τ_2).

Proof. Since, $\tau_1\tau_2$-$fsker(\tilde{1}_E) = \tilde{1}_E$ and $\tau_1\tau_2$-$fsker(\tilde{0}_E) = \tilde{0}_E$, then $\tilde{1}_E, \tilde{0}_E \in \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$.
Let $g_E, h_E \in \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$. Then, $\tau_1\tau_2 fsker(g_E) = g_E$ and $\tau_1\tau_2 fsker(h_E) = h_E$.
Therefore, by Theorem 3.3(6) , we have $\tau_1\tau_2 fsker[g_E \tilde{\cap} h_E] \tilde{\subseteq} g_E \tilde{\cap} h_E$,
but $g_E \tilde{\cap} h_E \tilde{\subseteq} \tau_1\tau_2 fsker[g_E \tilde{\cap} h_E]$, then $\tau_1\tau_2 fsker[g_E \tilde{\cap} h_E] = g_E \tilde{\cap} h_E$.
Hence, $g_E \tilde{\cap} h_E \in \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$.
Finally, let $\{g_{i_E} : i \in \Delta\} \in \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$. Then, $\tau_1\tau_2 fsker(g_{i_E}) = g_{i_E}$ for all $i \in \Delta$, it follows that $\tau_1\tau_2 fsker[\{\tilde{\cup}(g_{i_E}) : i \in \Delta\}] = \tilde{\bigcup}\{\tau_1\tau_2 fsker(g_{i_E}) : i \in \Delta\} = \tilde{\bigcup}\{g_{i_E} : i \in \Delta\}$.
Hence, $\tilde{\bigcup}\{g_{i_E} : i \in \Delta\} \in \tau_1\tau_2$-$\Lambda$-$FSS(X, \tau_1, \tau_2)_E$.
Consequently, the class $\tau_1\tau_2$-Λ-$FSS(X, \tau_1, \tau_2)_E$ is a fuzzy soft topology on X. □

Definition 3.4. Members of $\tau_{12\Lambda}$ are called $\tau_1\tau_2$-Λ - fuzzy soft open sets. A fuzzy soft set g_E in a fuzzy soft bitopological space (X, E, τ_1, τ_2) is called a $\tau_1\tau_2$-Λ-fuzzy soft closed set if its complement is a $\tau_1\tau_2$-Λ- fuzzy soft open set. We denote the family of all $\tau_1\tau_2$-Λ- fuzzy soft closed sets by $\tau_{12\Lambda}^c$.

Corollary 3.9. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then,*

$$\tau_i \subseteq \tau_{12} \subseteq \tau_{12\Lambda} \subseteq \widetilde{(X, E)}\ , \qquad i = 1, 2.$$

Remark 3.3. The following Example shows that $\tau_{12} \subseteq \tau_{12\Lambda} \subseteq \widetilde{(X, E)}$ in general.

Example 3.10. *Let $X = \{x, y\}, E = \{e_1, e_2\}$ and let*
$\tau_1 = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}\}$,
where
$g_{1_E} = \{g_1(e_1) = \{x/0.5, y/0.7\}, g_1(e_2) = \{x/0.3, y/0.6\}\}$,
$g_{2_E} = \{g_2(e_1) = \{x/0.4, y/0.5\}, g_2(e_2) = \{x/0.2, y/0.3\}\}$,
$f_{1_E} = \{f_1(e_1) = \{x/0.3, y/0.5\}, f_1(e_2) = \{x/0.3, y/0.6\}\}$,
$f_{2_E} = \{f_2(e_1) = \{x/0.4, y/0.5\}, f_2(e_2) = \{x/0.4, y/0.9\}\}$.
Then (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.
It is clear that $\tau_{12} = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}, f_{1_E}, f_{2_E}, p_E, q_E\}$, where
$p_E = g_{1_E} \tilde{\cup} f_{2_E} = \{p(e_1) = \{x/0.5, y/0.7\}, p(e_2) = \{x/0.4, y/0.9\}\}$,
$q_E = g_{2_E} \tilde{\cup} f_{1_E} = \{q(e_1) = \{x/0.4, y/0.5\}, q(e_2) = \{x/0.3, y/0.6\}\}$ *, while*
$g_{1_E} \tilde{\cup} f_{1_E} = f_{1_E}, \quad g_{2_E} \tilde{\cup} f_{2_E} = f_{2_E}$ *and*
$\tau_{12\Lambda} = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}, f_{1_E}, f_{2_E}, p_E, q_E, h_E\}$ *, where*
$h_E == \{h(e_1) = \{x/0.4, y/0.5\}, h(e_2) = \{x/0.3, y/0.4\}$.
It is clear that $\tau_{12} \subseteq \tau_{12\Lambda} \subseteq \widetilde{(X, E)}$.

Lemma 3.11. *An arbitrary intersection of $\tau_1\tau_2$-Λ-fuzzy soft sets is a $\tau_1\tau_2$-Λ-fuzzy soft set.*

Proof. Let $\{g_{i_E} : i \in \Delta\} \tilde{\subseteq} \tau_{12\Lambda}$. Then, $\tau_1\tau_2 fsker(g_{i_E}) = g_{i_E}$ for all $i \in \Delta$. From Theorem 3.3(6) we have $\tau_1\tau_2 fsker[\tilde{\bigcap}\{g_{i_E} : i \in \Delta\}] \tilde{\subseteq} \tilde{\bigcap}\{\tau_1\tau_2 fsker(g_{i_E}) : i \in \Delta\}$. , then $\tau_1\tau_2 fsker[\tilde{\bigcap}\{g_{i_E} : i \in \Delta\}] \tilde{\subseteq} \tilde{\bigcap}\{g_{i_E} : i \in \Delta\}$.
On the other hand, from Theorem 3.3(2) we have $\tilde{\bigcap}\{g_{i_E} : i \in \Delta\} \tilde{\subseteq} \tau_1\tau_2 fsker[\tilde{\bigcap}\{g_{i_E} : i \in \Delta\}]$. Hence, $\tau_1\tau_2 fsker[\tilde{\bigcap}\{g_{i_E} : i \in \Delta\}] = \tilde{\bigcap}\{g_{i_E} : i \in \Delta\}$. Therefore, $\tilde{\bigcap}\{g_{i_E} : i \in \Delta\} \tilde{\in} \tau_{12\Lambda}$. ☐

Definition 3.5. A fuzzy soft topological space (X, E, τ) is said to be an Alexandroff fuzzy soft space if the arbitrary intersection of fuzzy soft open sets is a fuzzy soft open set.

Theorem 3.12. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then, the fuzzy soft topological space $(X, E, \tau_{12\Lambda})$ associated of (X, E, τ_1, τ_2) is an Alexandroff fuzzy soft space.*

Proof. Immediate from the Theorem 3.3 and Lemma 3.11. ☐

Theorem 3.13. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then,*

(1) $\tilde{0}_E, \tilde{1}_E$ are $\tau_1\tau_2$-Λ-fuzzy soft closed sets.

(2) an arbitrary intersection of a $\tau_1\tau_2$-Λ-fuzzy soft closed sets is a $\tau_1\tau_2$-Λ-fuzzy soft closed set.

(3) an arbitrary union of a $\tau_1\tau_2$-Λ-fuzzy soft closed sets is a $\tau_1\tau_2$-Λ-fuzzy soft closed set.

Proof. Straightforward. ☐

Definition 3.6. A fuzzy soft set $g_E \tilde{\in} \widetilde{(X, E)}$ is said to be a $\tau_1\tau_2$-λ-fuzzy soft closed set in a fuzzy soft bitopological space (X, E, τ_1, τ_2) if $g_E = f_E \tilde{\cap} h_E$, where f_E is a $\tau_1\tau_2$-fuzzy soft closed set and h_E is a $\tau_1\tau_2$-Λ-fuzzy soft open set .

The family of all $\tau_1\tau_2$-λ-fuzzy soft sets will be denoted by $\tau_1\tau_2$-λ-$FSS(X, \tau_1, \tau_2)_E$.

Theorem 3.14. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then,* *(1) every $\tau_1\tau_2$-fuzzy soft closed set is a $\tau_1\tau_2$-λ-fuzzy soft closed set.*

(2) every $\tau_1\tau_2$-Λ-fuzzy soft closed set is a $\tau_1\tau_2$-λ-fuzzy soft closed set.

Proof. Straightforward. □

Corollary 3.15. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then, the following diagram is holds.*

$$\tau_1\tau_2 FSO \Rightarrow \tau_1\tau_2\Lambda FS \Rightarrow \tau_1\tau_2\lambda FCS \Leftarrow \tau_1\tau_2 FSC$$

Remark 3.4. The converse of Theorem 3.14 is not true in general which is shown in the following Example:

Example 3.16. *Let (X, E, τ_1, τ_2) is the same fuzzy soft bitopological space as in Example 3.2 and let $g_E = \{g(e_1) = \{x/0.4, y/0.5\}, g(e_2) = \{x/0.4, y/0.4\}$,*
Then
(1) g_E is a $\tau_1\tau_2$-λ-fuzzy soft closed set, because $g_E = p^c_E \tilde{\cap} f_{2_E}$ where p^c_E is a $\tau_1\tau_2$-fuzzy soft closed set and f_{2_E} is $\tau_1\tau_2$-Λ-fuzzy soft closed set [i.e. $\tau_1\tau_2 fsker(f_{2_E}) = f_{2_E}$]. But g_E is not $\tau_1\tau_2$-fuzzy soft closed set.
(2) It obvious that g_E is a $\tau_1\tau_2$-λ-fuzzy soft closed set, but it is not $\tau_1\tau_2$-Λ-fuzzy soft closed set, because $\tau_1\tau_2$-$fsker(g_E) = f_{2_E} \tilde{\cap} \tilde{1}_E = f_{2_E} = \{f_2(e_1) = \{x/0.4, y/0.5\}, f_2(e_2) = \{x/0.4, y/0.9\}\} \neq g_E$

Theorem 3.17. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $g_E \tilde{\in} \widetilde{(X, E)}$. Then, the following statements are equivalent:*

(1) g_E is a $\tau_1\tau_2$-λ-fuzzy soft closed set.

(2) $g_E = h_E \tilde{\bigcap} \tau_1\tau_2 cl(g_E)$, $\quad h_E \in \tau_{12\Lambda}$.

(3) $g_E = \tau_1\tau_2 fsker(g_E) \tilde{\bigcap} \tau_1\tau_2 cl(g_E)$

Proof. (1) $\Rightarrow$ (2): Let g_E be a $\tau_1\tau_2$-λ-fuzzy soft closed set. Then, there exists $m_E \in \tau^c_{12}, h_E \in \tau_{12\Lambda}$ such that $g_E = m_E \tilde{\cap} h_E$
$\Rightarrow g_E \tilde{\subseteq} m_E$
$\Rightarrow \tau_1\tau_2 cl(g_E) \tilde{\subseteq} \tau_1\tau_2 cl(m_E) = m_E$
$\Rightarrow h_E \tilde{\cap} \tau_1\tau_2 cl(g_E) \tilde{\subseteq} h_E \tilde{\cap} m_E$
$\Rightarrow h_E \tilde{\cap} \tau_1\tau_2 cl(g_E) \tilde{\subseteq} g_E$.

(2) $\Rightarrow$ (1): Since $g_E \tilde{\subseteq} \tau_1\tau_2 cl(g_E)$, then $h_E \tilde{\cap} g_E \tilde{\subseteq} h_E \tilde{\cap} \tau_1\tau_2 cl(g_E)$, but $g_E \tilde{\cap} h_E = g_E$, then $g_E \tilde{\subseteq} h_E \tilde{\cap} \tau_1\tau_2 cl(g_E)$. Therefore, $g_E = h_E \tilde{\cap} \tau_1\tau_2 cl(g_E)$, $h_E \in \tau_{12\Lambda}$. Hence (2) holds.

(2) $\Rightarrow$ (3): Since $g_E \tilde{\subseteq} \tau_1\tau_2 cl(g_E)$ and $g_E \tilde{\subseteq} \tau_1\tau_2 fsker(g_E)$, then $g_E \tilde{\subseteq} \tau_1\tau_2 cl(g_E) \tilde{\cap} \tau_1\tau_2 fsker(g_E)$. Now by (2), there exists $h_E \in \tau_{12\Lambda}$ such that $g_E = h_E \tilde{\cap} \tau_1\tau_2 cl(g_E)$
$\Rightarrow g_E \tilde{\subseteq} h_E$
$\Rightarrow \tau_1\tau_2 fsker(g_E) \tilde{\subseteq} \tau_1\tau_2 fsker(h_E)$ by Theorem 3.3(3)
$\Rightarrow \tau_1\tau_2 fsker(g_E) \tilde{\subseteq} h_E$, for $h_E \in \tau_{12\Lambda}$
$\Rightarrow \tau_1\tau_2 fsker(g_E) \tilde{\cap} \tau_1\tau_2 cl(g_E) \tilde{\subseteq} h_E \tilde{\cap} \tau_1\tau_2 cl(g_E)$
$\Rightarrow \tau_1\tau_2 fsker(g_E) \tilde{\cap} \tau_1\tau_2 cl(g_E) \tilde{\subseteq} g_E$.
Consequently, $g_E = \tau_1\tau_2 fsker(g_E) \tilde{\cap} \tau_1\tau_2 cl(g_E)$.
$Hence (3) holds.$

(3) $\Rightarrow$ (1): Since $\tau_1\tau_2 cl(g_E) \in \tau^c_{12}$ and $\tau_1\tau_2 fsker(g_E) \in \tau_{12\Lambda}$ by Theorem 3.3(5), then by (3) and Definition 3.6 we have that g_E is a $\tau_1\tau_2$-λ- fuzzy soft closed set. □

4 Conclusion

In this paper, we have presented characterizations of some types of fuzzy soft sets in fuzzy soft bitopological spaces and then presented its some basic properties with some examples. This paper can be improved for the studies on fuzzy soft bitopological spaces. In the future, using these sets, various classes of mappings on fuzzy soft bitopological space can be studied. We hope that this study will be useful for research in theoretical as well as in a applicable directions.

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Competing Interests

Author has declared that no competing interests exist.

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