Asian Research Journal of Mathematics
2(2): 1-14, 2017; Article no.ARJOM.30643

SCIENCEDOMAIN *international*
www.sciencedomain.org

Dynamics Response of Beam on Elastic Foundation with Axial Force to Partially Distributed Moving Loads

S. O. Sangoniyi[1] and F. O. Akinpelu[2*]

[1]Department of Mathematics, Emmanuel Alayande College of Education, Oyo, Nigeria.
[2]Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria.

Authors' contributions

This work was carried out in collaboration between two authors. Author SOS designed the study, wrote the protocol and as well wrote the first draft of the manuscript. Author FOA managed the literature searches, analyzed the study and interpreted the results of the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/30643
Editor(s):
(1) Gongxian Xu, Associate Professor, Department of Mathematics, Bohai University, Jinzhou, China.
Reviewers:
(1) Ayhan Goktepe, Selcuk University, Konya, Turkey.
(2) Pierre R. Blanquet, Univ. P & M Curie, Quai St Bernard, Paris, France.
Complete Peer review History: http://www.sciencedomain.org/review-history/17598

Original Research Article

Received: 22nd November 2016
Accepted: 11th January 2017
Published: 24th January 2017

Abstract

The dynamics response of Beam on elastic foundation with axial force to partially distributed moving loads was examined. The fourth order partial differential equation which is the governing equation was first reduced to second order differential equation by assume a solution in form of series solution and maple software was finally employed to obtain numerical solution in order to determine the behaviour of the system of discussion. The effect of mass of the beam on both the moving force and the moving mass were examined. It was noted that increase in mass of the beam is an increase in the transverse displacement for both the moving force and the moving mass with respect to both the tensile and the compressive forces. Again, it was observed that as the axial force increases, the displacement decreases under the action of the moving force but reverse is the case for the moving mass in that, as the axial force increases the displacement is equally increases for the tensile force but for compressive force, the displacement increases as the axial force increases. More so, it was discovered that deflection due to compressive force is greater than that due to tensile force. Furthermore, it was discovered that the displacement decreases as the elastic foundation (k) increases for both the moving force and the moving

**Corresponding author: E-mail: foakinpelu@yahoo.co.uk;*

mass for fixed value of M and S with respect to tensile force and compressive force. Finally, the deflection due to compressive force is greater than that due to tensile force while the deflection due to moving mass is greater than that due to moving force.

Keywords: Force; axial force; partially distributed; elastic foundation and moving loads.

1 Introduction

The Dee bridge disaster which was a rail accident caused tremendous human loss in England on 24th May, 1847 in Chester. Then after, there was such occurrence tagged "Collapse of Tay Bridge "on 28th December, 1879 in Britain. This was reported to be the worst structural failure to have occurred in terms of both lives lost and the size of the failed structure. Several books have been written on the said disaster by [1,2,3] and this is in no doubt attracted the attention of many researchers in the field of civil engineering, physics and applied mathematics to have interest in doing findings on the dynamic beam structures carrying moving loads on them.

Akinpelu [4] examined the response of viscously damped Euler-Bernoulli beam to uniform partially distributed moving loads. Here, it was observed that the beam has more than one mode of vibration with each mode having a different natural frequency. Concerning the result, it was discovered that as the mass of the load increases the amplitude is also increases and the value of the magnification factor occurs for a value ω less than one as $\varepsilon = 0.1$.

Akinpelu and Sangoniyi [5] studied dynamics response of simply supported axial force Euler-Bernoulli beam subjected to partially distributed moving load. In this study, the effect of axial force and mass of the beam on the moving force were examined with respect to both the tensile and the compressive forces. It was observed that as the mass of the beam increases, the transverse displacement is also increases for both the tensile and the compressive forces under the actions of the moving force and that of the moving mass. It was equally observed that as the axial force is increasing, the displacement is decreasing for the tensile force for the moving force while in the case of compressive force, as the axial force increases, the displacement is also increases for both the moving force and the moving mass.

Dobromir dinev [6] considered the analytical solution of beam on elastic foundation by singularity functions. The basic methods for solving the governing equations ware considered and their advantages and disadvantages were analyzed.

Michaltsos [7] examined the parameters affecting the dynamic response of light (steel) bridges. The author was able to bring together and examined a lot of parameters that are usually not taken into account either during the design of a bridge or because some assumptions were supposed by the designer to hold true for the design and the calculations of a bridge.

Dynamic Behaviour of a double Rayleigh Beam-System due to uniform partially distributed Moving Load was examined by Gbadeyan and Agboola [8]. The solutions of the problem formulated for the beams subjected to a uniform partially distributed moving load were found by applying the Fourier and Laplace integral transform techniques. It was found that the maximum amplitudes of deflection of the beams increases as the speed of the moving load increases. It was also discovered that the maximum amplitude of deflections of the upper beam increases while that of the lower beam decreases due to the increase of the rotary inertia. Finally, it was also found that the amplitudes of deflection of both beams increase with increase in the fixed length of the load.

The Effects of linearly varying distributed moving loads on beams was considered by Gbadeyan and Dada [9]. The study examined the problem of assessing the dynamic behaviour of simply supported undamped Bernoulli-Euler beams under a linearly varying distributed moving load. The governing equation for the mathematical model was analytically simplified into a set of ordinary differential equations that are solved by using Duhamel integral.

Adetunde et al. [10] investigated the dynamic analysis of non pre-stressed Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving load. The authors' modeled the problem mathematically in such a way that the moving load is small compared with mass of the beam. It was observed from the result that; the amplitude deflection decreases as the length of the load (ε) increases for a fixed value of the moving load (M_L)

Chopade and Barjibhe [11] considered free vibration analysis of fixed free beam with theoretical and numerical approach. The authors placed their focus on the analysis of transverse vibration of fixed free beam and also investigated the mode shape frequency. It was concluded that theoretical data is in good agreement with numerical results with negligible error.

Gbadeyan et al. [12] investigated Dynamic Response of Two Viscoelastically Connected Rayleigh Beams Subjected to Concentrated Moving Load. The governing differential equations of the problem were solved by using the finite Fourier Integral and Laplace transformation techniques with respect to the prescribed initial and boundary conditions. The effects of the velocity of the moving load, rotary inertia of the beam, stiffness coefficient and damping coefficient of the joining layer on the deflections of the beams were also highlighted. It was found that the amplitude of the deflection of both the upper and lower beams increases with increasing values of velocity of the moving load.

Nawal [13] considered free vibration of simply supported beam subjected to axial force. The author examined the free transverse vibrations of a uniform beam that is simply supported at both ends subjected to a static axial force. The beam considered for this study are of different shapes of cross section, that is, Rectangle section, Box section, I - section and T – section with the same value of area for all but different values of second moment of area. The effects of static axial force (tension and compression) on the characteristics of vibration were considered. It was shown that the beam of rectangular cross section and that of T – section have the least and the highest natural frequency respectively when compared with other types for the same applied force. José [14] examined the dynamic loads in new engineering codes for railway bridges in Europe and Spain.

Ogunyebi and Sunday [15] worked on the response of a non -uniform beam transverse by mobile distributed loads. The author investigated the response of non-uniform beam under tensile stress and resting on an elastic foundation. The fourth order partial differential equation governing the problem was solved when the beam is transverse by mobile distributed loads. The elastic properties of the beam, the flexible rigidity, and the mass per unit length were expressed as functions of the spatial variable using Struble's method. It was observed that the deflection of non-uniform beam under the action of moving masses is higher than the deflection of moving force when only the force effects of the moving load was considered.

Omolofe and Ogunyebi [16] considered the transverse vibrations of elastic thin beam resting on variable elastic foundations subjected to traveling distributed forces. The robust techniques called Galerkin's method in conjuction with integral transform method were used to treat the fourth order partial differential equations describing the motion of the beam load system. It was equally found that the incorporating axial force N, foundation modulus K and a damping term in the governing equation of motion increase the critical velocity of dynamical system, thereby reducing the risk of resonance.

You-Qi Tang et al. [17] studied parametric resonance of axially moving Timoshenko beams with time-dependent speed. The Hamilton principle was employed to obtain the governing equation which is a nonlinear partial-differential equation due to the geometric nonlinearity caused by the finite stretch of the beam. The method of multiple scales was applied to predict the steady-state response. The stability of straight equilibrium and nontrivial steady-state response were analyzed by using the Lyapunov linearized stability theory. Some numerical examples were presented to demonstrate the effects of speed pulsation and the nonlinearity in the first two principal parametric resonances.

Obviously, when the distributed loads are moving on a structure, they produce greater deflection and stresses than when such loads are static, thus, the analysis of the behaviour of beam under moving loads are of great importance to those in the field of structural engineering, physics and applied mathematics, hence, there is need for the study of moving load on elastic foundation which is of great importance in the field of transportation.

This paper examined the simply supported dynamics response on elastic foundation axial force Euler-Bernoulli beam subjected to partially distributed moving load and the main objectives of the study are stated thus:

i. To reduce the fourth order partial differential equation to second order differential equation.
ii. To solve the second order differential equation analytically using maple software.
iii. To examine the effects of mass of the load on the displacement response of simply supported beam under the action of moving force and moving mass.
iv. To determine the effects of axial force on the moving force and the moving mass.
v. To compare the displacement response of simply supported beam due to moving force and moving mass.
vi. To make comparison between the displacement response of simply supported beam with tensile and compressive forces.

2 The Governing Equation

Dynamics response of Beam on elastic foundation with axial force to partially distributed moving loads was considered and the governing equation is given as equation (1) which is:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y}{\partial x^2}-Sy\right)+m(x)\frac{\partial^2 y}{\partial t^2}+ky=F(x,t) \tag{1}$$

The boundary and initial conditions are:

$$\begin{Bmatrix} y(0,t)=y^{''}(0,t)=0 \\ y(L,t)=y^{''}(L,t)=0 \end{Bmatrix} \tag{2}$$

$$y(x,0)=y^{'}(x,0)=0 \tag{3}$$

Where; V is the constant velocity of the load, Ɛ is the length of the beam, E is the young modulus, I is the moment of inertial, EI is the flexural rigidity, y is the deflection/transverse displacement, x is the Coordinate, t is the time, M is the mass of the load or beam, m is the mass per unit length, g is the acceleration due to gravity, H is the Heaviside unit function, ξ = v t + $\frac{\varepsilon}{2}$ is the fixed length of the load, S is the axial force, k is the elastic foundation and F(x, t) is the applied force. This applied force per unit length $F(x,t)$ is the uniform partially distributed moving load which is given as

$$EI\frac{\partial^4 y}{\partial \mathrm{x}^4}+\mathrm{m}\left(\frac{\partial^2 y}{\partial t^2}\right)-sy\frac{\partial^2 y}{\partial x^2}+ky=F(x,t) \tag{4}$$

3 Method of Solution

Substitute for the applied force F (x, t) into equation (1) to have equation (5) as:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y}{\partial x^2}-Sy\right)+m(x)\frac{\partial^2 y}{\partial t^2}+ky=\left(Mg-M\left(\frac{\partial^2 y}{\partial t^2}+2v\frac{\partial^2 y}{\partial x\partial t}+v^2\frac{\partial^2 y}{\partial x^2}\right)\right)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right) \qquad (5)$$

This fourth order partial differential equation in (5) has to be reduced to second order ordinary differential equation and the assumed solution is as follows:

$$\text{Y(x, t)} = \sum_{i=1}^{n} Y_i(x)T_i(t) \qquad (6)$$

When eq. (6) is used on eq. (5) then;

$$EI(x)\sum_{i=1}^{n}Y_i^{iv}(x)T_i(t)-S\sum_{i=1}^{n}Y_i^{''}(x)T_i(t)+m(x)\sum_{i=1}^{n}Y_i(x)\ddot{T}_i(t)+k\sum_{i=1}^{n}y_i(x)T_i(t)=$$
$$\left(Mg-M\sum_{i=1}^{n}Y_i(x)\ddot{T}_i(t)+2MvY_i^{'}(x)\dot{T}_i(t)-mv^2Y_i^{''}(x)T_i(t)\right)\left(H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})\right) \qquad (7)$$

Multiply equation (7) by y_j (x) and integrate along the length of the beam and applying the orthogonality condition gives

$$EI\sum_{i=1}^{n}\int_0^L y_i^{iv}(x)T_i(t)y_j(x)dx+s\sum_{i=1}^{n}\int_0^L y_i^{ii}(x)T_i(t)y_j(x)dx+m(x)\sum_{i=0}^{n}\int_{\circ}^{L} y_i(x)\ddot{T}_i(t)y_j(x)dx+k\sum_{i=1}^{n}y_i(x)T_i(t)y_i(x)dx=$$
$$Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_j(x)dx-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i(x)y_j(x)dx-2Mv\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{'}(x)y_j(x)dx-Mv^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{''}(x)y_j(x)dx \qquad (8)$$

As per the free vibration, $EIy^{iv}=m\omega^2 y$ then equation (8) becomes

$$m\omega^2T_i(t)+sT_i(t)+m(x)\ddot{T}_i(t)+kT_i(t)=$$
$$Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_j(x)dx-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i(x)y_j(x)dx-2Mv\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{'}(x)y_jdx-Mv^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{'}(x)y_jdx \qquad (9)$$

But $y_i(x)$ with respect to elastic foundation gives;

$$Y_i(x)=\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi x}{L}\right) \qquad (10)$$

Determine the derivative of eq. (10) and substitute in (9) to have

$$m(x)\ddot{T}_i(t) + \left(m\omega^2 + S + k\right)T_i(t) = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi}{L}\right)dx$$

$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi}{L}\right)\right)dx$$

$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi}{L}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi x}{L}\right)\right)dx$$

$$+MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi}{L}\right)^2\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi x}{L}\right)\right)dx \tag{11}$$

Let $P = \left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}$, $Q = \left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}$ and applying trigonometry identity on equation (11) so that

$$\ddot{T}_i(t)+(\omega^2+\frac{s}{m}+\frac{k}{m})T_i(t) = -\frac{Mg}{m}\sqrt{\frac{2}{L}}\left(\frac{L}{Qj\pi}\right)\left[\cos\left(Qj\frac{\pi x}{L}\right)\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}$$
$$-\frac{2M}{2mL}\sum_{i=1}^{n}\overset{\circ\circ}{T}_i(t)\left[\left(\frac{L}{P\pi(i+j)}\right)\sin\left(P\frac{\pi}{L}(i+j)x\right)-\frac{L}{Q\pi(i-j)}\sin\left(Q\frac{\pi}{L}(i-j)x\right)\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}$$
$$-\frac{4MV}{2mL}\left(P\frac{i\pi}{L}\right)\sum_{i=1}^{n}\dot{T}_i(t)\left[\frac{-L}{P\pi(i+j)}\cos\left(P\frac{\pi}{L}(i+j)x\right)+\frac{L}{Q\pi(i-j)}\cos\left(Q\frac{\pi}{L}(i-j)x\right)\right]_{\xi-\frac{\varepsilon 15}{2}}^{\xi+\frac{\varepsilon}{2}} \tag{12}$$
$$-\frac{2MV^2}{2mL}\left(\frac{pi\pi}{L}\right)^2\sum_{i=1}^{n}T_i(t)\left[\frac{L}{p\pi(i+j)}\sin\left(P\frac{\pi}{L}(i+j)x\right)-\frac{L}{Q\pi(i-j)}\sin\left(Q\frac{\pi}{L}(i-j)x\right)\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}$$

Further simplification and substitution of P and Q in eq. (12) gives

$$\ddot{T}_i(t)+\left(\omega^2+\frac{S}{m}+\frac{k}{m}\right)T_i(t) = \frac{\sqrt{8L}}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{\frac{1}{4}}j\pi}Mg\sin\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi}{L}\xi\sin\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{\frac{1}{4}}\frac{j\pi}{2L}\varepsilon$$

$$-\frac{2M}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{\frac{1}{4}}\pi}\sum_{i=1}^{n}\ddot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{\frac{1}{4}}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{\frac{1}{4}}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{\frac{1}{4}}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{\frac{1}{4}}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$
\begin{aligned}
&+\frac{4Mvi}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left(\begin{aligned}&\frac{1}{(i-j)}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i-j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\right)\\
&-\frac{1}{(i+j)}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i+j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\right)\end{aligned}\right)\\
&+\frac{2Mv^2\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}i^2\pi}{mL^2}\sum_{i=1}^{n}T_i(t)\left(\begin{aligned}&\frac{1}{(i-j)}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i-j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\right)\\
&-\frac{1}{(i-j)}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i+j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\right)\end{aligned}\right)
\end{aligned}
\tag{13}
$$

4 Numerical Analysis and Discussion of Results

Two cases were considered as far as this paper is concerned which are a case of moving force and a case of moving mass.

4.1 The moving force

In this case, only the first term on the right hand side of the equation (13) is retained as the inertia effect of the moving load is neglected, hence equation (14) is obtained

$$
\ddot{T}_i(t)+\left(\omega^2+\frac{S}{m}+\frac{k}{m}\right)T_i(t)=\frac{\sqrt{8L}}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}j\pi}Mg\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{j\pi}{L}\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{j\pi}{2L}\varepsilon\right)
\tag{14}
$$

4.2 The moving mass

Here, both the inertia effect of the moving load and the force effect were considered in (13) resulted into equation (15) as follows;

$$
\begin{aligned}
&\left[1+\frac{2M}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\pi}\sum_{i=1}^{n}\left(\begin{aligned}&\frac{1}{(i-j)}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i-j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\right)\\
&-\frac{1}{(i+j)}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i+j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\right)\end{aligned}\right)\right]\ddot{T}_i(t)\\
&-\frac{4Mvi}{mL}\sum_{i=1}^{n}\left(\begin{aligned}&\frac{1}{(i-j)}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i-j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\right)\\
&-\frac{1}{(i+j)}\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i+j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\right)\end{aligned}\right)\dot{T}_i(t)
\end{aligned}
$$

$$+\left[\left(\omega^2+\frac{s}{m}+\frac{k}{m}\right)-\frac{2MV^2\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4} i^2\pi}{mL^2}\sum_{i=1}^{\infty}\frac{1}{(i-j)}\left(\begin{array}{l}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i-j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\right)\\ -\frac{1}{(i-j)}\cos\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{L}(i+j)\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right]^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\right)\end{array}\right)\right]T_i(t)$$

$$=$$

$$\frac{\sqrt{8L}}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}}Mg\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4} j\frac{\pi}{L}\xi\right)\sin\left(\left[1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right]^{1/4}\frac{j\pi}{L}\varepsilon\right) \tag{15}$$

Eq. (14) and (15) were solved numerically by the help of maple software using the following numerical values:

ε = 0.1, m = 70, V=3.3, g = 9.8, π =3.142, $\lambda = \frac{n\pi}{2}$, ε = 0.1, E = 2.07x10^{11}, I = 12.27x10^{-6}, m = 70, k = 50000, 150000 and 450000. $\omega = \sqrt{\frac{\lambda^4 EI}{m}}$, L=10, α = 10, θ = 0.1, $\xi = vt + \frac{\varepsilon}{2}$

Figs. 4.1 to 4.4 are the graphs of displacement against time for moving force and moving mass for the tensile and the compressive forces with various values of mass M. It was observed that the displacement increases as M increases for both the tensile and the compressive forces for the moving force and the moving mass.

Figs. 4.5 to 4.8 are the graphs of displacement against time for moving force and moving mass for both the tensile and the compressive forces with various values of axial force, S. Figs. 4.5 and 4.6 shown that displacement decreases as S increases for both the tensile and the compressive forces but for the case of the moving mass, it was revealed that as S increases, the displacement decreases for the tensile force as reflected in Fig. 4.7 while for the compressive force, the displacement increases as S increases in Fig. 4.8.

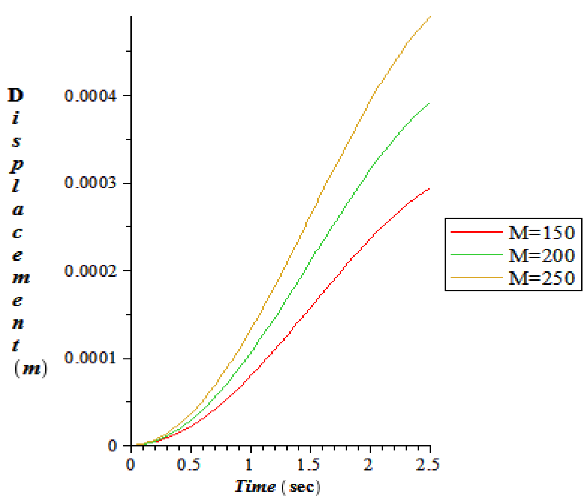

Figure 4.1: Displacement against time for moving force for a constant value of K with tensile force for various values of M

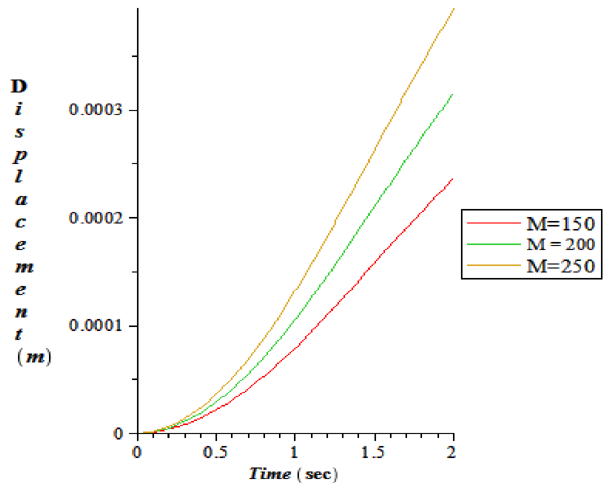

Figure 4.2: Displacement against time for moving force for a constant value of K with compressive force for various values of M for box shape

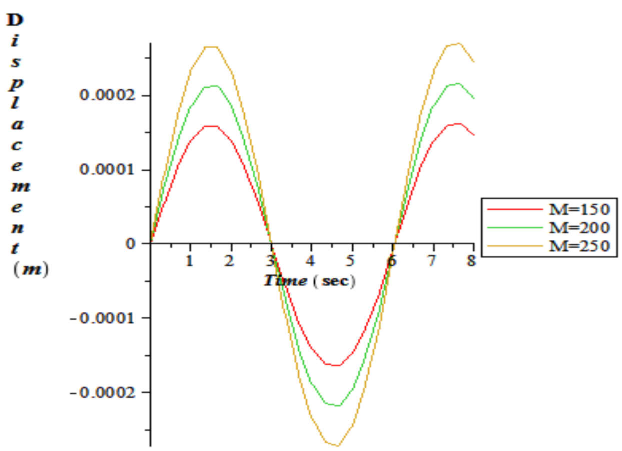

Figure 4.3: Displacement against time for moving mass for a constant value of K with tensile force for various values of M

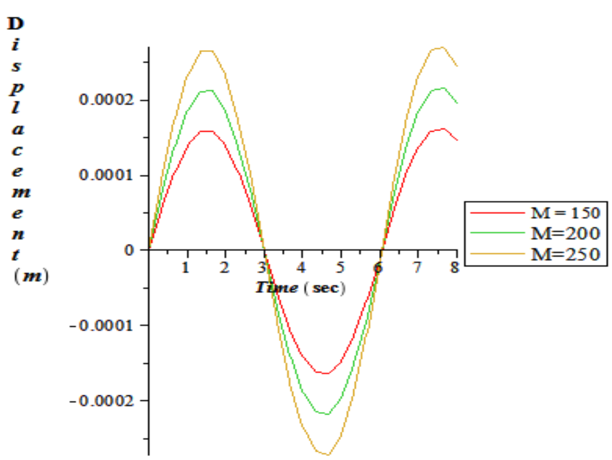

Figure 4.4: Displacement against time for moving mass for a constant value of K with compressive force for various values of M

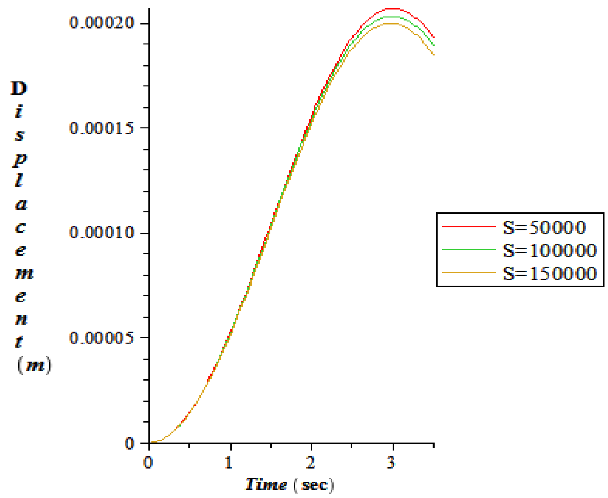

Figure 4.5: Displacement against time for moving force for a constant value of K with tensile force for various values of S

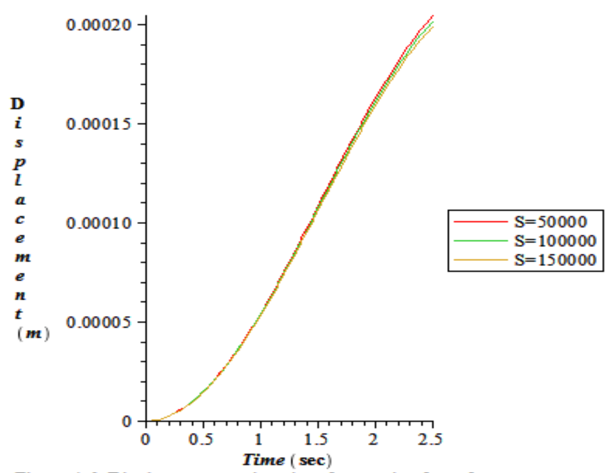

Figure 4.6: Displacement against time for moving force for a constant value of K with compressive force for various values of S

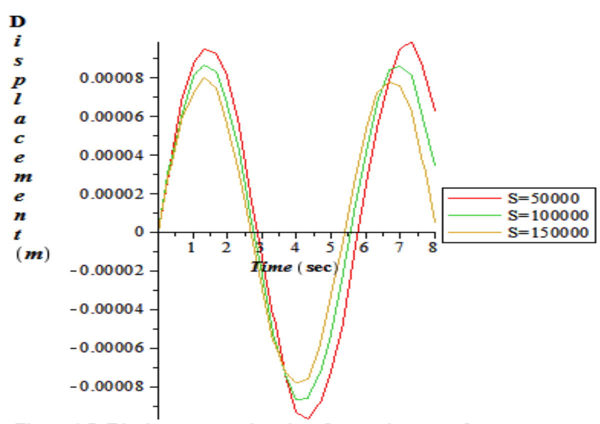

Figure 4.7: Displacement against time for moving mass for a constant value of K with tensile force for various values of S

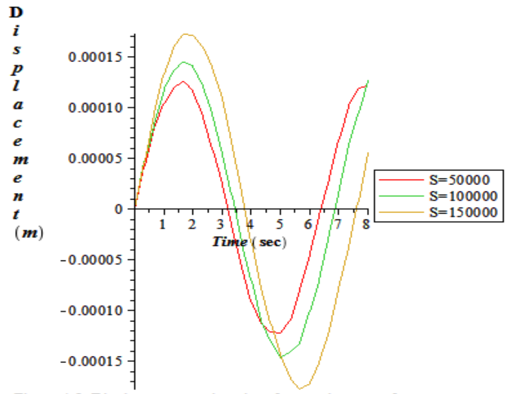

Figure 4.8: Displacement against time for moving mass for a constant value of K with compressive force for various values of S

Figs. 4.9 to 4.12 are graphs of displacement against time for moving force and moving mass for fixed values of M and S with tensile force and compressive force for various values of elastic foundation (K). It was discovered that the displacement decreases as the elastic foundation (k) increases for both the moving force and the moving mass with respect to tensile force and compressive force.

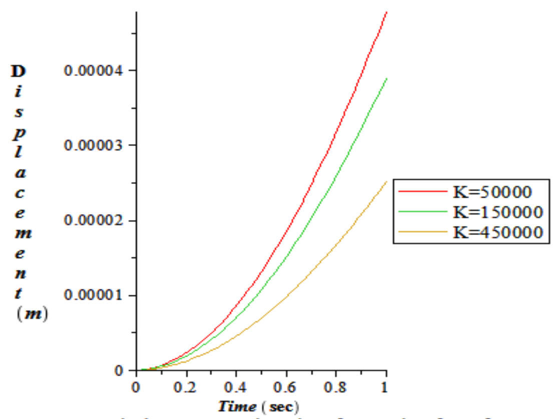

Figure 4.9: Displacement against time for moving force for a constant values of M and S with tensile force for various values of K

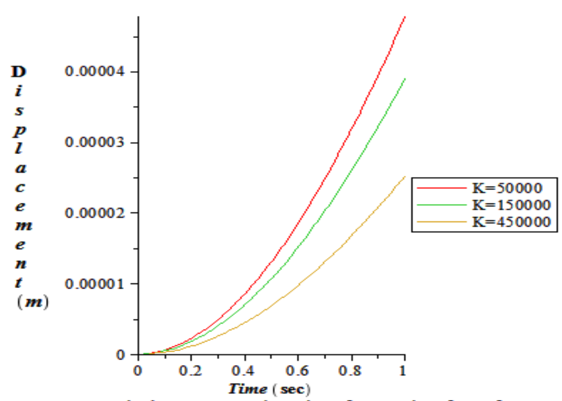

Figure 4.10: Displacement against time for moving force for a constant values of M and S with compressive force for various values of K

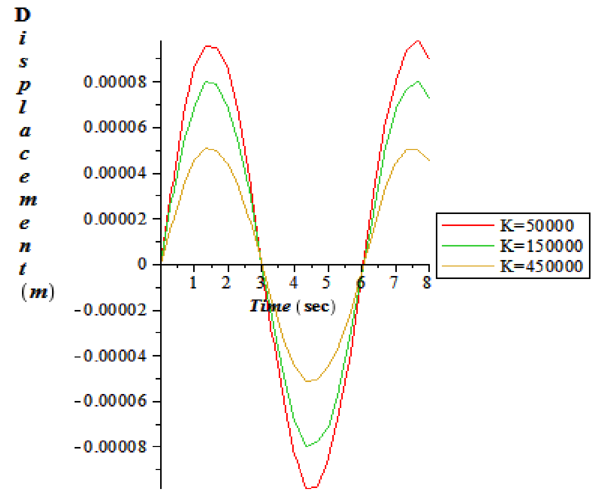

Figure 4.11: Displacement against time for moving mass for a constant values of M and S with tensile force for various values of K

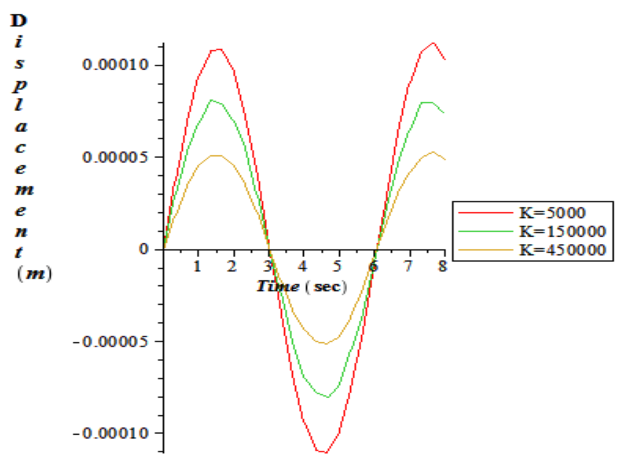

Figure 4.12: Displacement against time for moving mass for a constant values of M and S with compressive force for various values of K

Fig. 4.13 is displacement against time for both tensile and compressive forces with respect to mass. Here, it was shown that the deflection due to compressive force is greater than that due to tensile force. Fig. 4.14 is displacement against time for moving force and moving mass with respect to mass. This figure shown that the deflection due to moving mass is greater than that due to moving force.

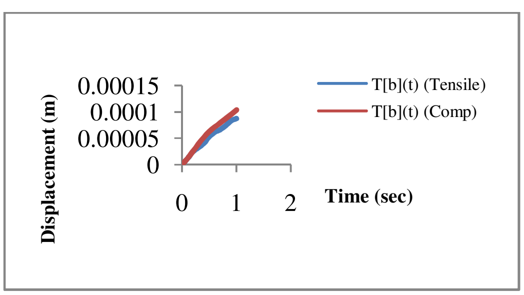

Fig. 4.13. Displacement against time for both tensile and compressive forces with respect to mass

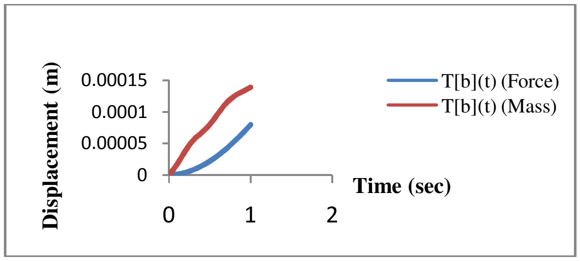

Fig. 4.14. Displacement against time for moving force and moving mass with respect to mass

5 Conclusion

Dynamics response of Beam on elastic foundation with axial force to partially distributed moving loads was investigated. The governing equation which was a fourth order partial differential equation was first reduced to second order differential equation by assume a solution in form of series solution and numerical method using maple software to examine the behaviour of the system of discussion. Mass of the beam on the moving force and the effect of axial force were examined with respect to the force (both tensile and compressive forces). It was noted that increase in mass of the beam resulted to increase in transverse displacement called deflection for both the moving force and the moving mass with respect to the tensile and the compressive forces. It was also observed that as the axial force increases, the displacement decreases under the action of the moving force while in the case of moving mass, as the axial force increases the displacement decreases for the tensile force but for compressive force, the displacement increases as the axial force increases. Again, for the case of displacement against time for moving force and moving mass for constant values of M and S with tensile force and compressive force for various values of K, it was discovered that the displacement decreases as the elastic foundation (k) increases for both the moving force and the moving mass. Finally, it was shown that the deflection due to compressive force is greater than that due to tensile force and in making comparison between the displacement response of simply supported beam due to moving force and moving mass, the deflection due to moving mass is greater than that due to moving force.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] David Swinfen. The fall of the Tay Bridge. Edinburgh, Mercat Press; 1994.

[2] John Thomas. The Tay Bridge disaster: New light on the 1879 tragedy. Newton Abbot, David and Charles; 1972.

[3] Matin T, Macleod I. The Tay Bridge disaster; A reappraisal based on modern analysis methods'; Proceedings of the institution of civil engineers. Civil Engineering, 1995, 77-83; more details of their method of analysis are given in 'Developments in structural engineering: Forth Rail Bridge centenary conference'; London. Spon; 1990.

[4] Akinpelu FO. The response of viscously damped Euler-Bernoulli beam to uniform partially distributed moving loads. Applied Mathematics. 2012;3:199-204.
Available:http://www.SCiRP.org/journal/am
DOI: 10.4236/am.2012.33031

[5] Akinpelu FO, Sangoniyi SO. Dynamics response of simply supported axial force Euler-Bernoulli beam subjected to partially distributed moving load. International Journal of Innovation in science and Mathematics. 2015;3(2):91–100.
ISSN: 2347-9051

[6] Dobromir Dinev. Analytical solution of beam on elastic foundation by singularity functions. Engineering Mechanics. 2012;19(6):381–392.

[7] Michaltsos GT. Parameters affecting the dynamic response of light (steel) bridges. The Scientific Journal Facta Universitatis. Series: Mechanics, Automatic Control and Robotics. 2000;2(10):1203–1218.

[8] Gbadeyan JA, Agboola OO. Dynamic behaviour of a double Rayleigh Beam-system due to uniform partially distributed moving load. Journal of Applied Sciences Research. 2012;8(1):571-581.

[9] Gbadeyan JA, Dada MS. The effects of linearly varying distributed moving loads on beams. Journal of Engineering and Applied Sciences. 2007;2(6):1006-1011.

[10] Adetunde IA, Akinpelu FO, Gbadeyan JA. Dynamic analysis of non prestressed Rayleigh Beam carrying an added mass and transverse by uniform partially distributed moving loads. Journal of Engineering and Applied Sciences. 2007;2(2):445-455.

[11] Chopade JP, Barjibhe RB. Free vibration analysis of fixed free beam with theoretical and numerical approach method. International Journal of Innovations in Engineering and Technology (IJIET). 2013;2:352–356.

[12] Jacob Gbadeyan A, Moses Dada S, Olasunmbo Agboola O. Dynamic response of two viscoelastically connected Rayleigh Beams subjected to concentrated moving load. The Pacific Journal of Science and Technology. 2011;12(1).
Available:http://www.akamaiuniversity.us/PJST.htm

[13] Nawal Al – Raheimy H. Free vibration of simply supported beam subjected to axial force. Journal of Babylon University/Engineering sciences. 2012;20(1):301-315.

[14] José Goicolea M, Gabaldón F, Domínguez J, Navarro JA. Dynamic loads in new engineering codes for railway bridges in Europe and Spain José m. Goicolea. Bridges for High-Speed Railways. 2008;43-58.

[15] Ogunyebi SN, Sunday J. On The response of a non -uniform beam transverse by mobile distributed loads. Global Journal of Science Frontier Research Mathematics & Decision Sciences. 2012;12(3). (Version 1.0 March 2012)

[16] Omolofe B, Ogunyebi SN. Transverse vibrations of elastic thin beam resting on variable elastic foundations and subjected to traveling distributed forces. The Pacific Journal of Science and Technology. 2009;10(2).
Available:http://www.akamaiuniversity.us/PJST.htm

[17] You-Qi Tang, Li-Qun Chen, Xiao-Dong Yang. Parametric resonance of axially moving Timoshenko beams with time-dependent speed. Springer Science+Business Media B.V. 2009, Nonlinear Dyn. 2009;58:715–724.
DOI 10.1007/s11071-009-9512-1

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/17598