







# Does Gravity Fall Down? Evidence for Gravitational-wave Deflection along the Line of Sight to GW170817

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## Abstract

We present a novel test of general relativity (GR): measuring the geometric component of the time delay due to gravitational lensing. GR predicts that photons and gravitational waves follow the same geodesic paths and thus experience the same geometric time delay. We show that for typical systems, the time delays are tens of seconds, and thus can dominate over astrophysical delays in the timing of photon emission. For the case of GW170817, we use a multi-plane lensing code to evaluate the time delay due to four massive halos along the line of sight. From literature mass and distance measurements of these halos, we establish at high confidence (significantly greater than  $5\sigma$ ) that the gravitational waves of GW170817 underwent gravitational deflection to arrive within 1.7 s of the photons.

*Unified Astronomy Thesaurus concepts:* [Weak gravitational lensing \(1797\)](#); [Gravitational waves \(678\)](#); [Gravitational deflection \(663\)](#); [Gravitational wave sources \(677\)](#); [Gravitational wave astronomy \(675\)](#)

## 1. Introduction

1.734 ± 0.054 s after the LIGO/Virgo detection of the gravitational waves from GW170817, Fermi and *International Gamma-ray Astrophysics Laboratory* observed the arrival of gamma-rays (Abbott et al. 2017a, 2017b). This near-simultaneous arrival of gravitational waves and photons over 130 Myr of travel time provides a strict test for modified gravity theories (Baker et al. 2017; Langlois et al. 2018). Boran et al. (2018) estimate that the Shapiro delay (Shapiro 1964) was ∼400 days (see also Abbott et al. 2017b), enabling further tests of general relativity (GR).

Recently, Mukherjee et al. (2019a, 2019b) have proposed measuring gravitational lensing of gravitational waves as a new probe of GR. They estimate that this lensing may be detectable in the future. In this work, we propose performing the same test, but using the geometric component of the lensing time delay as a test of GR. If gravitational waves and photons are both subject to the same geometric deflection, then they undergo the same lensing amplification. Likewise, if gravitational waves and photons undergo the same geometric deflection, then they should require the same travel time.<sup>4</sup> Investigating the possibility that gravitational waves undergo *larger* deflection than photons requires constraining the intrinsic time delay between gravitational-wave emission and photon emission. For example, gravitational waves could be emitted 100 s ahead of the photons, but delayed by 100 extra s due to larger deflection for near-simultaneous arrival. Considering this possibility is beyond the scope of this work.

Section 2 shows that, for typical nearby (tens of Mpc) gravitational-wave sources, the geometric component of the time delay is of the order of tens of seconds. Section 3 shows our computation of an approximate time delay for GW170817, obtaining a 68% confidence interval of 400–2200 s. Thus, we

show that the GW170817 gravitational waves must have been geometrically deflected line-of-sight halos by an amount at least comparable to photons to have arrived at nearly the same time. We summarize and conclude in Section 4.

## 2. Single-lens Time-delay Estimate

We show an illustration of a single thin gravitational lens in Figure 1. For an axially symmetric thin lens (line-of-sight size much less than the line-of-sight distances), the lensing deflection is given by

$$\hat{\alpha} = \frac{4GM(\xi)}{c^2\xi} = 1.91 \times 10^{-7} \text{ rad} \left[ \frac{M/(10^{12}M_{\odot})}{\xi/\text{Mpc}} \right], \quad (1)$$

where  $\xi$  is the impact parameter, and  $M(\xi)$  is the enclosed mass at radius  $\xi$  (e.g., Schneider et al. 1992). For typical impact parameters of hundreds of kiloparsecs and galaxy-scale lenses, we can approximate  $M(\xi)$  as the total mass  $M$ . For nearby sources like GW170817 (redshift  $\sim 0.01$ ), we can neglect the expansion of the universe (setting  $z = 0$ ). The extra path length due to the geometric deflection from the lens is

$$\sqrt{D_{ds}^2 + (\xi - \eta)^2} + \sqrt{D_d^2 + \xi^2} - \sqrt{D_s^2 + \eta^2}. \quad (2)$$

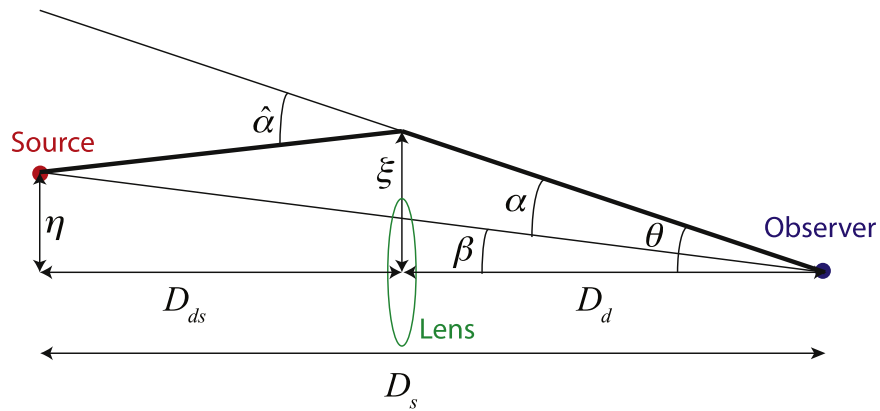
Assuming small angles so that we can substitute  $\beta = \eta/D_s$ ,  $\theta = \xi/D_d$ ,  $\alpha D_s = \hat{\alpha} D_{ds}$ , and also using  $\alpha + \beta = \theta$ , we expand Equation (2) to lowest order in  $\hat{\alpha}$  and  $\theta$  to find the extra path length is

$$\frac{1}{2} \frac{D_d D_{ds}}{D_s} \hat{\alpha}^2. \quad (3)$$

Thus the geometric component of the time delay is

$$\frac{1}{2c} \frac{D_d D_{ds}}{D_s} \hat{\alpha}^2 = 18.9 \text{ s} \left[ \frac{D_d D_{ds}}{D_s} \frac{1}{10 \text{ Mpc}} \right] \times \left[ \frac{M/(10^{12}M_{\odot})}{\xi/\text{Mpc}} \right]^2. \quad (4)$$

<sup>4</sup> With a sample size of one (just GW170817), it is extremely unlikely but perhaps possible that a conspiracy with different speeds for gravitational waves and photons could result in near-simultaneous arrival without gravitational waves undergoing the same geometric deflection as photons. This conspiracy would have to involve traveling through flat space (130 Myr), Shapiro delay ( $\sim 400$  days), and the extra time due to geometric deflection of the photons ( $\sim 800$  s; see Section 3). This possibility will be completely eliminated if a second event, necessarily having a different relative combination of these travel times, also shows near-simultaneous photon and gravitational-wave arrival.



**Figure 1.** Geometry of a thin gravitational lens in the observer–source–lens plane using similar notation as, e.g., Schneider et al. (1992). The bold line shows the path the light takes from source to observer.  $\hat{\alpha}$  is the deflection angle (Equation (1)), the impact parameter is  $\xi$ , the source is offset from the lens line of sight by a true (not observed) distance  $\eta$ ,  $D_d$  is the distance to the lens,  $D_s$  is the distance to the source, and  $D_{ds}$  is the distance from the lens to the source. For our derivation of a typical time delay in Section 2, we assume all angles are small, so  $\beta = \eta/D_s$ ,  $\theta = \xi/D_d$ , and  $\alpha D_s = \hat{\alpha} D_{ds}$ .

These time delays are of the order of the astrophysical time delay between gravitational waves and photons and will thus frequently be detectable, depending especially on  $M$  and  $\xi$  (both of which enter quadratically). It was pointed out to us by Nathan Johnson-McDaniel that a deflected ray passes farther from the lens and is thus subject to less Shapiro delay than a ray that passes straight through. Evaluating this difference in Shapiro delay in the small-angle and weak-field limits shows that it is twice the effect of the extra path length, and thus all of our computed time delays should be negative. Due to the small intrinsic time delay of  $\lesssim 10$  s (e.g., Abbott et al. 2017b), this oversight does not affect our conclusions.

### 3. Constraints from GW170817

To estimate the geometric time delay, we select galaxies with possible lensing contributions along the line of sight to GW170817 from the 2MASS Redshift Survey (Huchra et al. 2012). For each of the 43,533 2MASS galaxies included in the survey, we estimate the time delay up to a multiplicative constant from Equation (4). We estimate the distance from the cosmic microwave background (CMB)-centric redshift, the impact parameter from the distance and angular separation, and the mass from the absolute  $K$ -band isophotal magnitude. Coulter et al. (2017) discovered the GW170817 optical emission, necessary to obtain the coordinates on the sky and the identity of the host galaxy (NGC 4993). Cantiello et al. (2018) provide a precise distance to NGC 4993:  $40.7^{+2.5}_{-2.3}$  Mpc. Four galaxies are the most plausible for large time delays: NGC 5084, M104, M83, and NGC 5128 (Centaurus A). The later two of these are the central galaxies in groups and all have dynamical halo mass measurements.<sup>5</sup> To be conservative, we assume all dynamical mass measurements are at fixed distance. As estimated dynamical mass scales with estimated distance, in addition to the mass uncertainties discussed below, we assume

<sup>5</sup> An alternative to measuring total halo mass is to estimate the stellar mass through the luminosity and then use a measured halo mass/stellar mass relation (e.g., Behroozi et al. 2010) to estimate the halo mass. However, it is difficult to estimate masses to better than a factor of two with this approach, with roughly half coming from the stellar mass estimate (Conroy 2013), and half from the uncertainties of and scatter around halo mass/stellar mass relations (More et al. 2009; Behroozi et al. 2010). This leads to at least a factor of four uncertainty in time delay, as time delay scales as  $(\text{mass})^2$ . Furthermore, when estimating through absolute magnitudes there is an additional dependence on distance of stellar mass  $\propto (\text{distance})^2$ , as the distance is necessary to infer absolute magnitudes.

additional uncertainty on mass that covaries with the distance uncertainty.

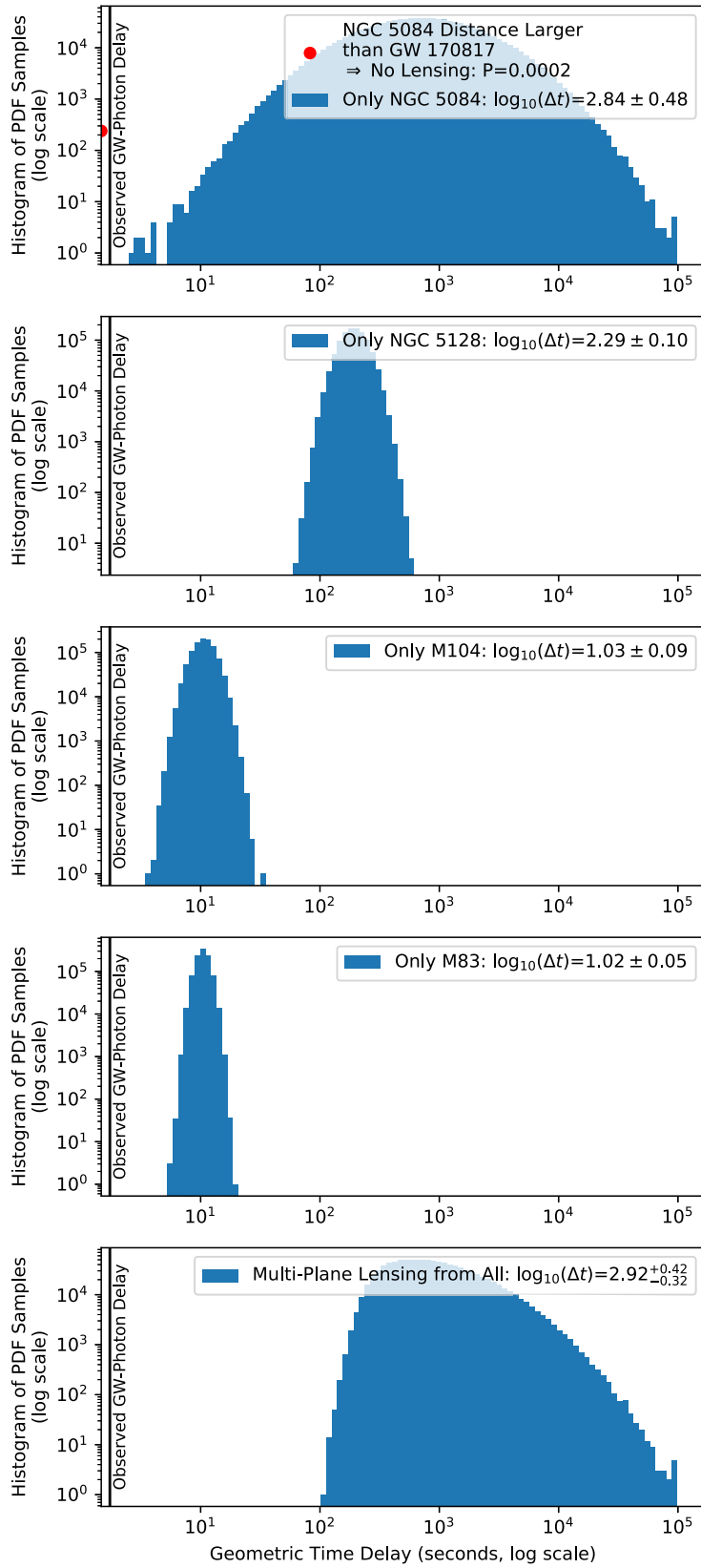
With a dynamical mass as large as  $10^{13}M_\odot$  (Carignan et al. 1997), NGC 5084 is one of the most massive disk galaxies known. For a more conservative mass measurement, we use the Carignan et al. (1997) value excluding two possible satellites that may not be bound:  $(5.2 \pm 2.9) \times 10^{12}M_\odot$ , which we convert to a  $\log_{10}(M/M_\odot)$  measurement of  $12.72 \pm 0.24$ . We take a distance modulus of  $31.12 \pm 0.54$  (or 16.7 Mpc; Springob et al. 2014), giving an impact parameter of  $0.84 \pm 0.24$  Mpc. We note that for this measured distance, there is a  $\sim 0.02\%$  chance that NGC 5084 is actually behind NGC 4993 and thus NGC 5084 could not lens GW170817.

M104 has a lower mass than NGC 5084, but it is more precisely measured. Tempel & Tenjes (2006) find  $2 \times 10^{12}M_\odot$ , in good agreement with the Jardel et al. (2011) value for 50 kpc. Jardel et al. (2011) measure to larger scales and find a higher mass ( $\sim 3 \times 10^{12}M_\odot$  with about 10% uncertainty). To be conservative, we use  $(2 \pm 0.2) \times 10^{12}M_\odot$ . We average two consistent tip of the red giant branch distance measurements to M104 (McQuinn et al. 2016 and the Extragalactic Distance Database; Jacobs et al. 2009) to arrive at a distance modulus of  $29.88 \pm 0.08$  (or 9.46 Mpc), giving an impact parameter of  $2.27 \pm 0.08$  Mpc.

Karachentsev et al. (2007) compute the tip of the red giant branch distances and the halo masses for the NGC 5128 and M83 groups. They find a mean distance of  $3.76 \pm 0.05$  Mpc for the NGC 5128 group (for an impact parameter of  $1.31 \pm 0.02$  Mpc) and  $4.79 \pm 0.1$  Mpc for the M83 group (impact parameter of  $0.74 \pm 0.02$  Mpc). Their mass for the NGC 5128 group is  $(6.4\text{--}8.1) \times 10^{12}M_\odot$ , which we convert to a  $\log_{10}(M/M_\odot)$  measurement of  $12.86 \pm 0.05$ . For the the M83 group, they find  $(0.8\text{--}0.9) \times 10^{12}M_\odot$ , which we convert to a  $\log_{10}(M/M_\odot)$  measurement of  $11.93 \pm 0.02$ . Of course, modeling these groups as axially symmetric masses (for the purposes of Equation (1)) is incorrect in detail. However, we show below that the implied time delays are large enough that moderate changes to our assumptions will not affect our conclusions.

#### 3.1. Combined Result

To properly evaluate the total geometric delay due to each of these four halos along the line of sight, we wrote a simple multi-plane lensing code. This code starts a ray at the position



**Figure 2.** Constraints on the geometric component of the time delay along the line of sight to GW170817. We draw 1000,000 realizations, varying halo masses and distances, then compute the time delay for each. As the constraints are nearly log-normal, we make histograms of the time delays in log-spaced bins. The top four panels show the time delays for each halo; the bottom panel shows the multi-plane lensing calculation for all halos. To investigate the tails of the distributions, we show the y-axis logarithmically. We show the observed time delay between gravitational waves and photons with a vertical line. Each of the 1000,000 computed time delays for the multi-plane combined result is well above the observed time delay, indicating a secure detection of the geometric component of the time delay for the gravitational waves.

(relative to us) of GW170817 propagating toward the origin. As the ray reaches the position of closest approach to each halo, it is deflected according to Equation (1). Due to the deflections during propagation, the ray will not pass through the origin. We thus iterate, adjusting the initial direction in the next iteration to account for the miss in the last iteration. As the deflections are  $\sim 10^{-7}$  radians, this iteration converges rapidly. The difference between an undeflected line from GW170817 and the path taken gives the time delay. As this relative difference scales as the deflection angles squared ( $\sim 10^{-14}$ ) and the precision limit for a double-precision (64-bit) floating point is  $\sim 10^{-16}$ , we improve the accuracy of our results by using higher-precision arithmetic through the Python decimal package.

We compute a the probability density function for lensing by numerically propagating all distance and mass uncertainties. We draw 1000,000 realizations for all distances and masses and compute the geometric component of the time delay for each realization. The resulting probability density functions are shown in Figure 2. The top four panels show the time delay considering each halo in isolation; the bottom panel shows the multi-plane combination. Except for NGC 5084 (which does not have a precise enough distance measurement to securely place it along the line of sight to GW170817), we see strong ( $>5\sigma$ ) evidence of a time delay larger than the observed delay for each of the other halos. We see even stronger evidence for the combination (bottom panel of Figure 2) where the minimum time delay out of the realizations is 112 s and the 68% confidence interval is 400–2200 s. We can also interpret our result in terms of constraints on the deflection angle of gravitational waves ( $\hat{\alpha}_{\text{GW}}$ ), assuming that the gravitational waves left no later than the photons and traveled at the same speed. Starting from the lower bound of our 99.9999% confidence interval (112 s), the gravitational waves were deflected by at least 110 s. Assuming Equation (1) applies to photons, we find

$$\hat{\alpha}_{\text{GW}} > \frac{3.96GM(\xi)}{c^2\xi}. \quad (5)$$

#### 4. Conclusion





This work proposes a novel test of GR: detecting the deflection of gravitational waves by a gravitational potential by evaluating the geometric component of the time delay due to the deflection. Any deflection more or less than predicted by GR will lead to a different time delay with respect to the photons. We show that typical galaxy halos give detectable time delays (of order tens of seconds) for gravitational-wave sources as close as tens of Mpc. We present an initial evaluation of the deflection the gravitational waves of GW170817 underwent due to four halos along the line of

sight. We see strong evidence that deflection did occur and GR passes our test.

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Matplotlib (Hunter 2007), Numpy (van der Walt et al. 2011), Python, SciPy (Jones et al. 2001).

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